Valuation and Cost of Capital

Chapter 4

Alternative Methods for Translating Risk into Value and Computing Cost of Capital

Methods for Translating Risk into Value and Computing Cost of Capital

Introduction – Value and Risk

The very beginning of this book discussed that valuation of any investment depends on just two things: (1) the expected level of future cash flows produced by the investment; and, (2) the risk profile associated with those cash flows. Expected cash flow can be estimated from economic analysis of value drivers such as forward prices, forecasted demand or capacity utilization and cost structure, issues that were addressed in earlier chapters and will be re-visited later. This chapter concentrates on the second aspect of valuation – determining how risk associated with various value drivers can be converted into a measure of value in monetary terms. Making the conversion of risk into value involves the most basic proposition in finance – all else equal (i.e. expected cash flow is the same), if something is riskier, its value is lower. Although the tradeoff between risk and return is one of the very few things we really do know in finance, defining risk and measuring how differences in risk affect value is among the most daunting problems in all of economics. This problem, which amounts to finding a rate of return that will appropriately compensate investors for risk, is further complicated because of changes in risk perceptions that can be sudden and virtually impossible to measure. During the financial crisis for example, increased perception of risk was a thought to be a major cause of the stock market crash; the increased risk perception was the one reason the banks stopped lending; and higher perceived risk surely caused stress and led to many couples having marital problems. Alan Greenspan, the once revered and now somewhat pitiful figure, attributed the housing bubble and subsequent stock market crash to changes in the ways investors quantify risk.[1]

To see how valuation involves quantifying risk associated with expected cash flow, consider the example of a risk free government bond. With no variation between expected and realized cash flow, measuring value is an easy task – we simply discount the known cash flows at the current risk free rate of interest to obtain a definitive measure of value. As virtually all investments (including long-term government bonds with a fixed nominal interest rate) are not risk-free, an issue present in almost every problem in finance ranging from the capital asset pricing model to bond valuation to options pricing formulas to financial statement analysis is determining how risk affects the cost of capital and value. Given the difficulty in measuring how risk affects value, you may think it would be nice if valuation always consisted of measuring certain cash flows and then discounting them at the risk free rate, especially after then extreme volatility that has experienced in financial markets in the past decade. In such a risk free world where expected cash flow equals realized cash flow, we would have definitive answers for valuation. However, if all investments were really risk free and the future really was certain, then the world would be very mundane; everything would be the same, everybody would be perfect and there would be no spice to life. More importantly, no company would be able to earn real profits for its investors through managing its activities in a superior manner. This interesting and challenging side of risk may be difficult to appreciate after the stock market falls by 50% or the oil price reaches $150 per barrel, but without the central issue of risk assessment, finance and economics would be boring.

[1] FT Times, 26 March, 2008.

Of the two parts to valuation, making the forecast of expected cash flow may seem to be the more important than assigning risk to the forecast which is generally accomplished through making an assumption for the cost of capital. In practice, cost of capital is often simply dictated by someone in the treasury department of a corporation and it is tempting to think the issue is not a vital factor relative to the many other things that must be assessed in a valuation analysis. For some investment decisions, if the cost of capital changes from 8% to 12%, selected investments may not change very much implying that more effort should be made in projecting cash flow than in measuring risk. The question of what cost of capital to apply to forecasted cash flow, however, is crucial in industries that are capital intensive, since such a very large component of the total value added represents return on capital. In these industries, problems in assessing risk through understating or overstating the cost of capital can have big effects on both the type and the amount of new investment. Because of the importance of cost of capital in influencing investment decisions, there are also important public policy implications as to the level of the cost of capital. Some of the most capital intensive investments in the world are renewable electricity facilities such as wind and solar power. If government policies are to encourage these investments, the manner in which policies affect cost of capital may be much more important than how the policies affect the level of expected cash flow. Similarly, supporting investment in capital intensive infrastructure such as roads and public transport in developing countries depends heavily on the perceived risk associated with those investments.

To illustrate the importance of risk analysis to capital intensive projects, consider the decision to invest in electricity generation facilities with different levels of capital intensity introduced above. These investments, in increasing order of their capital intensity, are a combined cycle gas plant, an on-shore wind farm, an off-shore wind farm and a solar facility. (When ranking the projects by capital intensity, the level of revenue divided by the amount of capital spent to build the project can be used as the criteria.) Capital intensity statistic is driven by the amount of operating expense relative to capital expenditures and the lifetime of a project – the longer the life and the lower the operating expense, the higher the capital intensity. The table below demonstrates that in terms of this measure of capital intensity, the solar project is almost ten times as capital intensive as the combined cycle plant. Given plausible variations in cost of capital assumptions (discussed later in the chapter and shown on the left hand side of the table) the total cost of the solar project varies by 46.2% when the cost of capital changes, while the same cost of capital range only affects the total cost of the combined cycle plant by 6.5% (numbers to the right of the cost of capital column represent the total amount of revenue per unit of electricity produced in order to meet the different cost of capital requirements.) The analysis confirms that, given a level of expected cash flow, small changes in the cost of capital can cause investment decisions to radically change for capital intensive investments, while the similar differences in cost of capital would have a minimal effect on the investment decision of the non-capital intensive investment[2].

	WACC	On-Shore Wind	Off-Shore Wind	Solar	Combined Cycle
First year Revenue/MWH and Cost of Capital Assumption					
	5.92%	114.81	159.57	462.52	87.94
	7.88%	137.44	182.75	580.19	91.27
	8.85%	143.49	190.14	626.86	92.34
	9.15%	146.00	193.15	639.61	92.71
	9.78%	151.87	198.43	676.01	93.65
Maximum Revenue		151.87	198.43	676.01	93.65
Minimum Revenue		114.81	159.57	462.52	87.94
Percent Difference		32.3%	24.4%	46.2%	6.5%
Capital Intensity		55.6%	53.6%	91.5%	11.7%

Why Measuring the Cost of Capital is so Difficult

The entire focus of most investment decisions -- be it signing a contract, selling assets, acquiring a

[2] These results come from analysis in a file named renewable template.xls included on the CD.

company or purchasing a stock, is derived from estimating future prices, forecasting growth in demand, predicting the cost structure of operations, considering the cost of new capital expenditure and evaluating other value drivers of expected cash flow. In theory, risks associated with these value drivers are incorporated into investment decisions through application of a discount rate that depends on risk of specific projects. This discount rate that reflects risk is the cost of capital. For many reasons, finding the appropriate cost of capital to apply to projects with all sorts of different risks is an extremely complicated and some may say almost an impossible task.

To demonstrate the type of issues that make measuring risk so difficult, consider the case of valuing of alternative generating technologies in the electricity industry. For decades before deregulation was imagined, people in the industry debated how differences in risk affect the value of alternative coal, nuclear, gas, hydro and renewable generation technologies. There was no way that traditional financial tools which calculated cost of capital from the CAPM or from the dividend discount model could realistically be used in the problem of assessing risk of construction cost over-runs, fuel price volatility and demand growth uncertainty that resulted in excess capacity. Some argued that, not withstanding the very high up-front capital cost of nuclear power, these investments were beneficial because the price of nuclear fuel is not as volatile as the prices of oil, gas and coal necessary to run fossil fuel plants. Others argued that the high operating leverage and the uncertain capital cost risk of nuclear power meant that these plants had far more risk than other technologies. Although the topic was vigorously debated around the water cooler and in regulatory commission hearings, a rigorous and objective quantification of the difference in risk between a nuclear plant and a fossil plant was never really accomplished. These days, risk differences between renewable technologies and non-renewable technologies and investments in developing versus industrialized countries raise similar important policy questions. Analytical techniques that can potentially resolve questions such as these are presented in this chapter.

One of the main ideas of this chapter is that once cash flow projections are established, the task of somehow quantifying how cash flow risk associated with value drivers affects the cost of capital cannot really be boiled down to a simple formula. In technical terms, converting risk into value involves comparing the benefits from making an investment and receiving future cash flows with the opportunity costs of making another investment that has similar risks. If you could really find an investment with similar risk, then you could try to ascertain the minimum rates of return required by investors who make that similar investment. After finding this number from returns required for similar investments, you could apply that return criteria to expected cash flow from your investment. Should expected cash flow from your investment produce a lower rate of return than this minimum level acceptable for other investments, then you should invest in the other investments instead of your investment; that is the idea of opportunity cost. This notion of finding the minimum rate of return that is acceptable for an investment given the risk of the investment is the definition of cost of capital. The minimum return necessary to take risk, or the cost of capital, could be for expected cash flow received by debt investors; it could be for expected cash flow accruing to equity investors; or it could be for cash flow received by holders of other securities. A big problem with finding this level of minimum required return to compensate for risk is that the whole concept of risk itself is a very amorphous concept – measuring risk is something like measuring happiness. Because the subject of cost of capital involves trying to somehow put a quantitative value on risk which cannot even easily be defined, some of the discussion in this cannot be programmed in excel but instead involves more theoretical concepts than other parts of the book.

The cost of equity capital and not the cost of debt capital is normally the main subject discussed when addressing cost of capital discussion in business schools and finance texts. The most common methods applied in attempting to measure the cost of equity capital are the CAPM or the dividend discount model which use information from current or historic stock prices in deriving a number. The general idea is that since value is derived from cash flow and the cost of capital, and since stock prices measure the value of equity, one can try to back out the cost of capital from stock price data. To see how this process works in general, consider a very simple equation for the value of a stock shown below that assumes continual growth in cash flow (the idea of deriving cost of capital from stock value is discussed further in Appendix 2):

Stock Value = Equity Cash Flow/(Cost of Capital minus Cash Flow Growth Rate)

The problem with applying this idea to find the cost of capital is that while one can observe current stock prices, both the future growth in expected cash flow and the cost of capital are not known. Indeed, both the cost of capital and the growth rate are not constant and must be estimated indefinitely. This leaves you with one observation – the stock value -- and at the very minimum two unknowns – the expected period by period growth rate cash flow and the period by period cost of capital. As there are two unknowns with one equation for value, the implicit cost of capital can be measured only after an estimate is made of the future growth rate in cash flow. Since the future expected cash flow is not known (and cannot be estimated using highly biased and unreliable measures made by sell-side analyst expectations), the cost of capital is fundamentally unobservable.

Cost of capital is generally taught to MBA students all over the world using the CAPM equation which involves adding a risk premium to the risk free rate. A very difficult part of applying the CAPM is making an estimate of the rate or return required to compensate investors for taking risks in stocks rather than risk free bonds which is known as the equity market risk premium. This expected market risk premium number is virtually impossible to measure and in the end often boils down to market psychology from surveying opinions of various people or so-called sophisticated investors. Alternatively, it is sometimes estimated using a formula like the one shown above where estimated future cash flow growth and stock prices are used to back into the cost of capital number for each stock in the entire stock market. This derived cost of capital for each stock is averaged and subtracted from the risk free rate. In the end, if surveys are made or estimated growth rates are used, attempts to measure the current level of the risk premium either come from the subjective opinions of investors or they have the same problem of two unobservable variables and one equation discussed above.

In contrast to the cost of equity used for determining the risk of equity cash flow, when measuring the cost of debt capital, one can more easily observe the number. This is because the risk of not realizing expected debt cash flows (from interest and principal repayment) is the probability of default and the loss that is incurred when a default occurs. The opportunity cost of debt capital is therefore the required return on top of the risk free rate that investors need to earn to compensate for the risk that loans may default. Risk of default is traditionally measured by a credit rating or a credit score that explicitly or implicitly measures the likelihood of default. The minimum required rate of return that can be earned on debt to compensate for the probability of default is called the credit spread and it drives the cost of debt capital. Unlike the cost of equity capital which cannot be directly observed, the cost of debt capital can be observed through tabulating credit spreads. Simplified equations for the computing the value of debt can be expressed as:

Value of Debt = (Expected Cash Flow from Interest and Repayment)/(Cost of Debt Capital), or

Value of Debt = Cash Flow without Default/(Risk Free Rate plus Credit Spread)

Unlike the equation for equity cash flow, the expected level of debt cash flow is easy to compute which means that the cost of debt capital can be computed even if there is no data on the credit spread. In this case the expected cash flow and the value are both known. These formulas to value debt securities which are easier to implement than the above formula for equity investments are behind much of the cost of capital discussion in this chapter where debt capacity and the risk of debt are used as the foundation to find the overall risk of an investment.

Measurement of the cost of capital is further complicated in practice because investments in different securities or alternative physical facilities do not have similar risks to one another even the investments are in the same industry. For example, risk analysis often involves evaluating different tradeoffs between fixed and variable costs for alternative investments (such as a large fleet of small planes versus a small fleet of large planes) or must consider the value of a particular contract (such as project financing of pipeline projects) or changes in business strategy (such as measurement of synergies in mergers.) There is generally no way to find an exactly comparable investment and come up with the

minimum rate of return that defines risk in these situations. Further, the risk profile is changed for some investments that include more flexibility to cease operation or construction (which can be termed real options.) In working through different methods to value risk and evaluate the cost of capital, these issues as well as standard cost of capital problems must be addressed.

Cost of Capital and Three Valuation Techniques – CAPM, Debt Capacity and Simulation

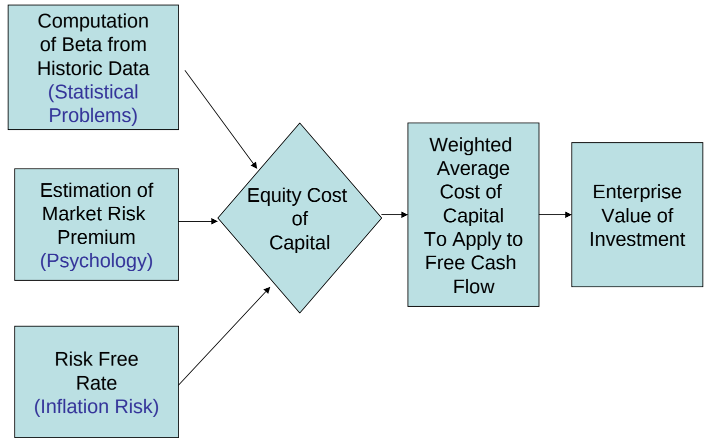
In sorting through the difficult problem of putting a value on risk, one can easily find a whole lot of writing in both academic texts and practical valuation books. Issues such as derivation of the CAPM and optimal capital structure are not discusses as they are addressed extensively in these texts. Instead, non-conventional ideas and practical modeling techniques that often conflict with Nobel Prize winning work are proposed. Unlike the traditional techniques, the alternative methods do not try to put risk into a single standard deviation or a beta statistic. Three distinct techniques are discussed in this chapter to measure cost of capital and convert risks into a measure of value. These include:

(1) Evaluating historic stock price data for cash flows with comparable risk using the CAPM;
(2) Using credit analysis and methods to evaluate loans to derive debt capacity in order to measure risk; or,
(3) Using the probability distribution of cash flows developed in simulations to measure risk.

The first approach for converting cash flow risk into value is measurement of cost of capital from the capital asset pricing model and weighted average cost of capital (“WACC”). (The use of free cash flow discounted at the WACC is very similar to the adjusted present value (“APV”) approach which derives the cost of equity capital as if no debt leverage were present.) The WACC/APV method relies on historic stock prices from comparable equity investments as the information base for measuring risk associated with things that will drive the future variation in future cash flow. When making a valuation from use of the CAPM, one first computes free cash flow that is independent of financing. Then one should theoretically discount that cash flow at a discount rate that also does not depend on the capital structure of the company as suggested by the famous Miller and Modigliani hypothesis.

As already discussed in Chapter 1 and elaborated on below, there are many practical and theoretical problems associated with these types of traditional models including the necessity of considering psychological factors in measuring risk premium and the lack of evidence that Beta (β) really measures firm specific risk. Problems with the CAPM and other stock price approaches range from theoretical problems with whether risk can be measured by the standard deviation of returns (notwithstanding the Nobel Prize winning work of Henry Markowitz[3]); to the lack of empirical data that beta measures risk (despite the Nobel Prize winning work of William Sharpe); to the impossibility of objectively measuring the expected market risk premium; to statistical problems with measuring beta. These problems are so serious that they imply alternative techniques to assess risk need to be considered when making investment decisions. A diagram of the process of using the CAPM to compute value is illustrated below.

[3] Economics Nobel Prizes have been awarded since 1969. The prizes for financial economics inclued1990 - Harry M. Markowitz, Merton H. Miller, William F. Sharpe; 1985 - Franco Modigliani and 1997 - Robert C. Merton, Myron S. Scholes.



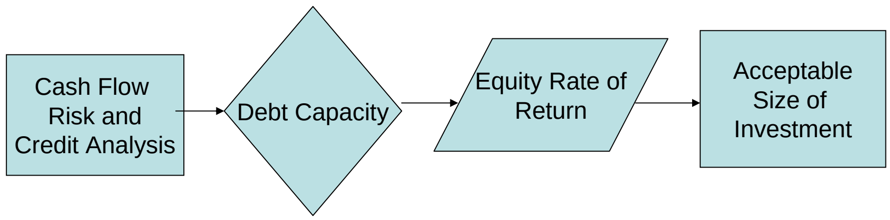
Given the problems with measuring beta and/or computing cost of capital using discounted dividend techniques, an alternative and relatively simple approach is presented which compares the market value realized for a stock with the amount of equity capital invested and retained in a company. One of the basic notions of value creation is that value is increased when a company earns more than its cost of capital. If a company is just earning its cost of capital, then the market value will be the same as the amount of equity investment meaning that if one can find investments with a market value equal to the amount invested, then by observing the earned return one can also find the cost of capital. This method of finding the cost of capital which is not taught in business schools and is not applied by investment bankers measures the minimum value that justifies an investment and evaluates the rate of return that is consistent with the minimum value.

To address very serious problems that arise with the CAPM/WACC methods, two alternative approaches that convert cash flows into value are presented in this chapter. These techniques are not taught in to MBA students and cannot be boiled down to a simple formula with three parameters as is the case with the CAPM. Instead, the approaches make use of the fact that valuation of debt is directly related to the overall value of an investment and that the cost of debt is a lot easier to calculate than the cost of equity. The two alternative approaches do not attempt to measure risk from historic stock price data, but rather derived the overall cost of capital and risk of an investment from forward-looking credit analysis.

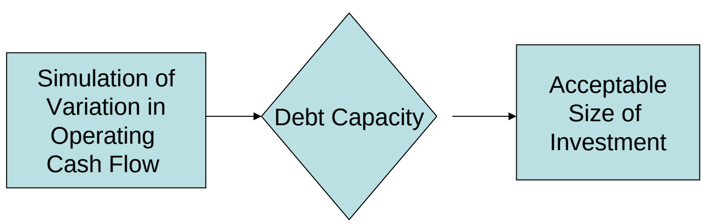
In many practical situations, investments are made only when debt can be raised to finance investment and they are not made when lenders will not provide funds. When lending stopped at the end of 2008, most capital investment also ceased and the worldwide economy fell into a tailspin – investment simply did not occur without debt. At the other end of the spectrum, when rating agencies did such a poor job in classifying risky investments like bundles of sub-prime loans as AAA, or rating Lehman Brothers as AA a week before the company defaulted, over-investment in housing and other industries occurred. These events and many others show that real world risk investment depends on the risk assessment of lenders as well as valuations made by equity investors. Finance professors and analysts who religiously apply the DCF method may still believe that the amount of debt that can be raised is irrelevant to the real world investment process (as implied by the Nobel prize winning work of Miller and Modigliani), but given the way lending influences capital investment, these people should no longer be taken very seriously.

Given that lending decisions clearly do affect investment decisions in practice, the second part of this chapter discusses the mechanics of how debt capacity can be used to translate risk into value and come up with an implicit coat of capital. The debt capacity approach differs from the CAPM and other traditional stock price methods because the information base for computing the risk associated with cash flow is credit analysis data from banks and credit rating agencies -- whose job it is to measure risk (even if they do their job very badly) -- rather than historic stock market data. In working through the debt capacity approach, techniques are presented that measure how lenders assess credit quality and

then how the debt capacity can be used to measure the overall value of an investment. Since the amount of debt that a lender will issue is limited by the risks present in operating cash flow, debt capacity analysis can be used to derive the overall risk and value of the investment, and not only the debt portion of the investment as shown in the diagram below. Risk analysis used to evaluate an investment then comes from bankers whose job it is to measure risk rather than somebody in the treasury department fiddling with beta statistics. One of the important benefits of this debt capacity approach that contrasts to the CAPM is that it allows one to directly compute differences in the risk and cost of capital associated with specific different construction and operating contacts and risks.



The third approach for translating risk into value applies mathematical analysis introduced in the last chapter to credit analysis. This method measures the value of an investment through simulating the probability distribution of future operating cash flow and then uses this probability distribution to compute debt capacity. One can think of this technique as building on the debt capacity approach but using time series parameters rather than judgments of bankers and credit analysts as the basis for credit analysis. Once debt capacity is derived, it is used to value the overall investment and the residual value to equity investors. Through simulating volatility in cash flow, risk to lenders and the value of debt is directly measured through computing the probability of default and the loss given default. The diagram below illustrates that the method begins with mathematical analysis that can in theory be independent of the poor decisions made in recent years by bankers and credit rating agencies (although inappropriate application of mathematical have been as discredited as much as the rating agencies.) As discussed in chapter 3, attempting to apply mathematical equations and normal distributions to economic variables that drive value can also be very dangerous.



The three alternative methods that are presented to quantify risk and measure value are summarized below. These approaches use different financial modeling techniques and the underlying information used in the analysis comes from completely different information sources. Traditional CAPM methods use stock market data, debt capacity approaches work from market information on financial criteria developed by bankers, and mathematical techniques apply information used to develop time series parameters.

Summary of Alternative Models Described to Compute Risk and Value			
Method	Information Source	**Analytical Technique**	**Problems with Model**
CAPM	Stock Prices on Comparable Firms and Overall Market Expectations	Compute Beta from Statistical Analysis	Data on Beta, Measurement of Market Premium, Theory, No Ability to Measure Specific Risks
Project Finance	Debt Leverage and Debt Service Coverage from Similar Transactions	Compute Cost of Capital from Equity Return Criteria and Maximum Debt Capacity	Gathering Data on Comparable Projects, Equity Hurdle Rate Assumption
Volatility Analysis	Volatility Statistics for Drivers of Value Such as Electricity Price	Compute Cost of Capital from Analysis of Yield Spread as an Option or from Variablility in Asset Value	Relationship of Debt to Equity or Relationship Between Variablilty and Value

Summary of Cost of Capital and Valuation Exercises

Analytical case exercises in this chapter apply the three different cost of capital techniques to translate risk into the value of an investment. These exercises include evaluating how a contract affects the value of an investment as well as how the values of different types of investments with varying capital intensity are affected by alternative cost of capital estimation approaches. The first exercise measures the value of different capital investments by using the traditional WACC/APV techniques and applying cost of capital measured with the CAPM. The second exercise values an investment using the debt capacity approach and illustrates how the approach can differentiate value depending on whether revenues are derived from spot prices versus long-term contracts with fixed prices. This case demonstrates that detailed risks associated with various contract provisions can be objectively quantified using credit analysis information that could not be assessed with the CAPM. The third exercise computes the required credit spread, debt capacity and overall value through applying Monte Carlo simulation techniques using alternative assumptions with respect to volatility and other parameters.

The various different financial models and risk measurement techniques presented below are used to derive discount rates that should be applied to free cash flow (analogous to the weighted cost of capital in traditional analysis.) These case studies demonstrate various problems and benefits with alternative approaches. When attempting to apply the CAPM in the context of an investment decision, the weighted average cost of capital varies by a wide margin because of ambiguities in the model. This wide range in cost of capital comes about because of difficulty in measuring beta and the impossibility of estimating the market risk premium. The fact that the range in cost of capital is large has dramatic implications on the decision to invest in capital intensive investments. Differences between the low range and the high range can easily change the whole investment decision for capital intensive assets. For example, when applying the lower end of the cost of capital range, investments which are capital intensive investment seems to be the optimal choice. On the other hand if one believes the upper end of the cost of capital range is more appropriate, the non-capital intensive asset appears to be best. If a model cannot even provide guidance as to the appropriate investment decisions, it is useless.

Method	Reason for Variation in Discount Rate	Low	High
CAPM	Uncertainty in Model of Beta and Rf	8.4%	13.6%
Project Finance	Contract Project vs Merchant Project	7.2%	11.4%
Time Series	Price Volatility of 10% and 20%	8.2%	12.2%

The second exercise uses equity cash flow in a project finance model and applies maximum debt capacity using information from market data on debt service coverage ratios. Unlike the first case in which the cost of capital model could not be used to make an investment decision, once the debt capacity is known and an estimate can be made of real world required returns required by equity investors, this method allows one to make an affirmative investment choice. Furthermore, application of the debt capacity approach can be used to measure risks of different strategies associated with an investment. For instance, using data from actual financings, the cost of capital is shown to be about 7.2% if cash flow risk is moderated with a long-term fixed price contract, while the cost of capital increases to 11.4% if cash flows are more subject to price volatility without a contract. Here, there is a

wide range in cost of capital estimates as with the first exercise, but this time the range explicitly measures differences in risk associated with a different contract strategy. Unlike the debt capacity approach, the CAPM could not be used to directly measure risk associated with a contract.

A third exercise demonstrates how the differences in the volatility of revenues can be translated into cost of capital and value. Using this approach, Monte Carlo simulation is applied in conjunction with a financial model to evaluate the distribution of cash flows and the probability of default on a loan. This case study illustrates that if 20% price volatility is assumed, the debt capacity and credit spread implies an overall cost of capital of 12.22%. In a case with 10% price volatility, the cost of capital derived from the credit spread falls to 8.2%. Through deriving the cost of capital from the distribution of cash flows, this method quantifies differences in value that arise from all of the time series analysis that was discussed in earlier chapters. Results of the three financial modeling case applications are shown on the table below:

Computing Cost of Capital Using the Capital Asset Pricing Model and Stock Market Data

The overall cost of capital can be thought of as including five components: (1) the real rate of interest in the economy that should be consistent with overall economic growth; (2) the expected rate of inflation in the currency of the expected cash flow; (3) the general average risk premium for risky investments compared to risk free investments across the whole worldwide economy; (4) the specific risk premium for the company related to operating factors assuming that it has no financial leverage; and (5) the added risk premium for the company from added leverage. The nominal risk free rate includes the first two of these and the inflation rate projected in the cash flow must be consistent with the expected risk free rate used in the model. For example, if cash flows are projected in Rupee, the expected inflation rate in the cash flow model should be consistent with the cost of capital.

In the past half century a seemingly elegant and well known formula, the CAPM, has been developed to measure cost of capital and account for all of the five factors discussed above. This model is probably familiar to most readers and some may wonder when it will be taught to ten year olds in grammar school. The basic proposition of CAPM is that the only risks which affect value are those that cannot be eliminated through diversification of a portfolio is derived from a 1952 paper by Harry Markowitz when he was a twenty-five year old graduate student at the University of Chicago. He suggested that risk can be quantified in terms of the standard deviation of returns and he showed that this standard deviation can be reduced through holding a wide variety of investments. The idea is that if you can eliminate risk by simply diversifying a portfolio, then you do not need to demand payment – in the form of a higher return – for accepting the risk.

The foundation of the CAPM is that risk can be measured by variability in cash flow and that cost of capital changes with the overall demand and supply of capital in the economy as well as the overall cost of borrowing at the risk free rate. Measurement of cost of capital depends therefore on only three factors – the risk free rate (R_f), the expected market premium above the risk free rate for all assets in the economy (EMRP or R_m), and the unique risk of an asset relative to other assets -- Beta (β). Most cost of capital analysis focuses on the unique risk of the company, the β, and development of a reasonable measure for the level of expected return (R_m) that is necessary for investors to accept risk in a portfolio of risky investments rather than risk free securities. To implement the CAPM, one adds a risk premium to the risk free rate of borrowing using the well known simple formula shown below. (In addition to these variables an adjustment is sometimes made for small companies and companies that operate in emerging markets.)

Cost of Equity Capital = Risk Free Rate ($R_{f,t}$) + Beta (β_i) x Expected Market Risk Premium (R_m)

Stepping back from the mathematics underneath the theory, the formula is intuitive. In measuring cost of capital you would generally like to add a risk premium to the nominal risk free rate and it is

reasonable that the risk premium should be the overall risk premium for all investments adjusted upwards or downwards to reflect the individual risk of the company. When the market crashes and the value of your stock falls by a lot less than the overall market, then it is logical to think the stock has relatively low risk. On the other hand, if the stock collapses by more than the overall market, you have a lot of risk.

The remainder of this section discusses how the CAPM can be applied in real world investment decisions. First, empirical problems with testing the underlying theory of the model are reviewed. Next issues with respect to the measuring each of the three parameters are discussed. The third section works through a practical case exercise on computing beta and the cost of equity. Fourth, the CAPM model is applied to a very capital intensive hydro project, a less capital intensive coal plant and an even less capital intensive natural gas plant. Fifth, a variation of the CAPM, the arbitrage pricing model, is discussed and applied to particular investments. The final section presents an alternative to the CAPM that uses statistical analysis on market to book ratios.

The general implication of the paragraphs below is that while the CAPM formula seems to be elegant and the theory has been accepted for decades, it is often useless in terms of making large scale investment decisions. The case exercise shows that the CAPM formula produces a wide range of cost of capital measures that does not allow one to hone in on the risk associated with a specific investment. Further, the model cannot be used for quantifying detailed components of an investment such as revenue, cost and construction contracts that may have a big effect on the overall risk of the investment. Finally, the analysis demonstrates that using different estimates of the market premium R_m and different statistical techniques to estimate the beta produces an unreasonably wide range in cost of capital estimates. When attempting to use these wide ranges to alternative investments, one is left with no definitive basis upon which to make a decision.

Empirical Studies of the CAPM and Theoretical Problems with the Model

Most of the discussion regarding the CAPM will work thorough practical difficulties in applying the model and measuring the three parameters. However, even if one could put reasonable parameters into the model, the CAPM would not be a valid way to translate risk into value because a large number of empirical studies of the model have not been able to prove that the model really works. This means that even if the beta and the equity market premium could be definitively established for an investment, if the theory itself is wrong, the results would still be meaningless. The fact that the underlying theory of the CAPM cannot be demonstrated on an empirical basis, but the model continues to be taught as a central component of study and applied in practice is remarkable. It would be as if a key component of medical science such as antibiotics is demonstrated to be useless, but the science of antibiotics continued to be taught in medical school and doctors did not change their practice.

To understand why the CAPM continues to be applied, it is instructive to compare the CAPM with the dividend discount model and its close cousin, the P/E valuation equation that were popular before the CAPM was used in a widespread manner. Before the CAPM began to be applied in the 1980's, companies would make judgments about the cost of capital or use a formula that backed out the cost of capital from the discounted value of future dividends known as the dividend discount model. The dividend discount model which is described along with derivation of valuation multiples in Appendix 1 to this chapter comes up with the implied cost of capital from using an analysis that marginal investors are assumed to apply in valuing shares when they buy and sell stocks. If the investors who are buying shares hold shares indefinitely and if one could somehow know what all future dividends will be, then, with knowledge of the expected future dividends, one can back-out the cost of equity the investors must have used in establishing the value of the stock. As explained in further detail below in the appendix, it is easy to derive a model of the value of a stock as:

$$P_0 = D_1/(\text{Cost of Equity} - \text{Constant Growth Forever})$$

If Growth, D_1 and P_0 are known, then:

$$\text{Cost of Equity} = D_1/P_0 + \text{Constant Growth Forever.}$$

When a company does not have a stable growth rate in dividends, but one thinks they know the growth rate in earnings, the future return on equity and the cost of equity, then a slightly more complex formula that depends on earnings growth and the ability to earn a return above the cost of capital can be applied as follows:

$$P_0/E_1 = (1\text{-Growth/Return on Equity})/(\text{Cost of Equity} - \text{Growth})$$

If one knows P_0/E_1 and can estimate ROE, Cost of Equity and Growth, then

$$\text{Cost of Equity} = (1\text{-Growth/Return on Equity})/(P_0/E_1) + \text{Growth}$$

The dividend discount and the P/E equation have many problems including requirements to make indefinite estimates of growth, return on equity and adjustments for changing dividend payout ratios. However, the model is simple explain and to apply once the growth rate is somehow estimated. Application of the model does not require deep thought and it is not very confusing. Further, the model confirms and obvious and underused fact that low growth stable companies have a lower cost of capital than high growth companies. When applying the discounted cash flow model in practice, the simple mistake that when growth is assumed to change the cost of capital should also change is almost always ignored.

In contrast to the dividend discount model and the P/E equation, when teaching the CAPM, finance professors could use statistics, utility functions, risk return frontiers, covariance analysis and other seemingly sophisticated concepts. Further, they could apply beta in event studies, evaluation of mutual fund returns and other applications to study a host of financial issues. Given the quick and widespread use of the model in business schools, many attempts were made to prove that the model really works in practice. Most tests of the CAPM boil down to the question whether beta is really the only relevant measure of risk although there are many other problems with measuring the expected market risk premium. As the basic proposition of finance is that increased risk is correlated with increased returns, at minimum the studies should demonstrate that higher betas come along with higher returns. Even this basic proposition could not be demonstrated. For example, in 1992, professors Eugene Fama and Ken French concluded:

> *In short, our tests do not support the most basic prediction of the [CAPM] that average stock returns are positively related to market betas.*[4]

The famous Fama and French paper was not the first study that could not validate the CAPM from a theoretical standpoint. Coplend, Koller and Murrin noted that the Fama and French study is the "most recent in a long line of empirical studies that questioned the usefulness of betas in explaining the risk premium." They summarize the Fama and French study as follows:

> *Fama and French concluded that equity returns are inversely related to the size of a company measured by the value of its equity capitalization, and positively related to the book value of the company's equity to its market value. When these variables were taken into account, beta added nothing to the ability to explain returns on equity.*[5]

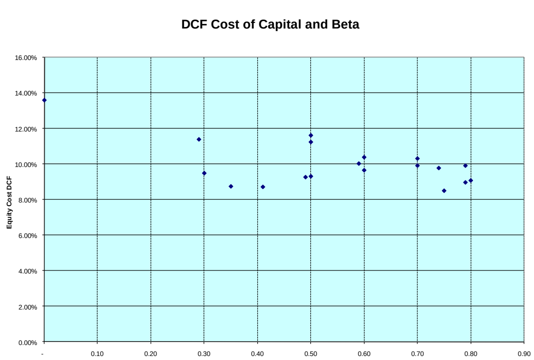
A more direct critique of portfolio theory and the assumption of returns that follow a normal distribution that underlies the CAPM is made by Nicholas Taleb:

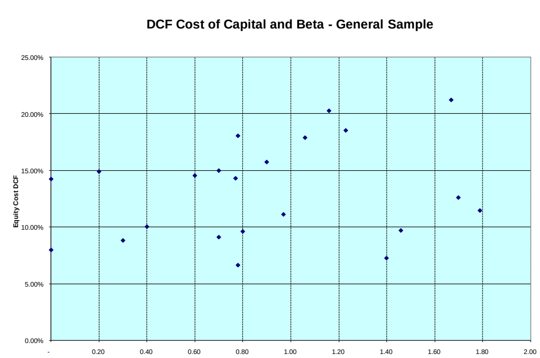
[4] E. Fama and K. French, "The Cross-Section of Expected Stock Returns," Journal of Finance (June 1992), pp. 427-465.
[5] T. Copeland, T. Koller and J. Murrin, *Valuation: Measuring and Managing the Value of Companies*, John Wiley and Sons, Inc. 2000. Page 224-225.

After the [1987] stock market crash [the Bank of Sweden] rewarded two theoreticians, Harry Markowitz and William Sharpe, who build beautiful models … contributing to what is called Modern Portfolio Theory. The … Sharpe and Markowitz models … work like quack remedies sold on the Internet …"[6]

The failure of empirical tests to verify the CAPM is not the only theoretical problems with the model. There are many other aspects of risk and valuation that CAPM and free cash flow that are not addressed in the model. Say you must decide between two types of projects in an industry -- different electricity plants, an on-shore versus and off shore oil project, alternative airplane types and routes, shopping centers versus sky scrapers or different sized factories. In these cases it is very likely that one project type has a different risk profile than the other. For example, assume the volatility of cash flow is twice as much for the one type of project – the off shore versus the on shore project -- in the same industry. Using the CAPM it would be virtually impossible to find betas that could differentiate the risks associated with the two different projects. Further, betas would not change when the structure of market changed as in the AES case, beta did not reveal the risk of speculative trading in the Constellation Energy case, beta would not measure the risk of erroneous traffic studies and consultant reports in the Eurotunnel case, beta did not reveal the risk associated with Enron's strategies (Value Line and Bloomberg both reported a beta of below 1.0) and betas were not relevant in measuring the risk of CDO investments that consisted of sub-prime mortgages. Finally, even if the beta is de-leveraged and re-leveraged, it cannot fully account for the effects of different debt levels in making and investment and the option characteristics of equity investment.

A rough test of the CAPM can be made by comparing the cost of capital from the Gordon model with the CAPM. If both the Gordon model and the CAPM model were reasonable ways to compute the cost of capital, then there should be a positive relationship between the cost of capital estimated from the Gordon model and the CAPM. Specifically, there should be a strong positive relationship between beta and the estimated cost of capital from application of the dividend discount model. The graph below uses betas published on the Finance.Yahoo! website along with cost of capital estimated from application of P/E equation for the same companies. To apply the P/E formula, forward returns on equity estimated by the stock analysts and five year estimated growth rates estimated by investment analysts are applied. The two graphs below show the analysis for a single industries and a selected group of companies across different industries. They demonstrate that there is virtually no relationship between beta and the estimated cost of capital. This simple study suggests that either the CAPM is wrong or the P/E analysis is wrong as there is not a strong positive relationship between the two cost of capital estimates. If the expected market risk premium in the CAPM is built using the dividend discount model as discussed below, then application of the CAPM requires both models to be correct. The simple analysis shown on the graphs below demonstrate that in this case the model cannot be valid.





Application of CAPM

[6] Taleb, Nassim, *The Black Swan: The Impact of the Highly Improbable,* Random House, 2007, Page 277.

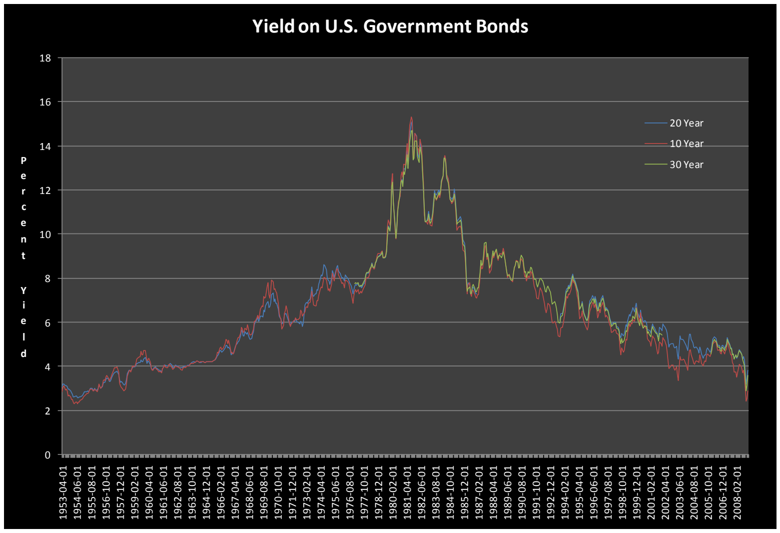
Pretending that the CAPM theory is valid and confirmed by empirical evidence, if the data for the model cannot be found or if inputs to the model are very ambiguous to estimate, then the model would still not useful in real world applications. Without the data to input into the CAPM, it is as if one is attempting to make a model of a new building, but the materials cannot be found. If materials cannot be found, the model of the building is then not useful in actually constructing the building. The paragraphs below discuss issues with the three parameters that must be input into the CAPM – the risk free rate, the beta and the market risk premium. This discussion is intended to accomplish two things. First, if you just want to know what people do in practice when computing cost of capital, this is part of the discussion. Second, working through each of the variables demonstrates how difficult it is to find objective data with each of the three input parameters -- $R_{f,t}$, β_i and R_m and just how hopeless the model is in practice.

Debates about the first parameter, the risk free rate ($R_{f,t}$) concern whether there truly is a risk free security that eliminates inflation risk and what duration of government bonds should be used as a proxy for risk free investments. Controversy surrounding the second parameter – beta (β_i) -- involves a host of statistical issues associated with whether relationships in past prices can be a reasonable reflection of prospective risk. Other practical issues with measurement of beta include the arbitrary and silly adjustment that is made to "raw" betas for mean reversion; whether daily, weekly or monthly data should be used in regression analysis; and how to adjust betas in a comparative sample when companies have different levels of leverage. The final parameter -- the expected market risk premium (R_m) -- is subject to even more dispute than the other input parameters. Questions involving how to measure the level of the expected market risk premium have been called "the most debated issue in finance"[7] and "the premier question relating to the cost of capital, for theorists and practitioners alike."[8] A large body of research demonstrates that use of historic data on the actual market returns relative to the actual risk free rate to measure the expected market returns is a biased approach that leads to over-estimation of the cost of equity.

Estimation of the Risk Free Rate ($R_{f,i}$)

The first and seemingly most straightforward parameter to be estimated in the CAPM is the risk free rate. A reading of articles and books on valuation and testimony presented in valuation litigation cases demonstrates that the most common way the model is applied is to use the interest rate on ten-year government bonds as a proxy for the risk free rate. In their well known book on valuation, Copeland, Koller and Murrin also suggest using the ten-year Treasury bond rate.[9] Since cash flows are typically expressed in nominal terms and since the opportunity cost of holding money is different for different time lengths, the bond term should be consistent with the holding period of equity. If inflation risk is to be avoided, one could buy an inflation indexed bond where the principle increases with the overall rate of inflation. This means the tenor of risk free bonds should be consistent with the duration of equity cash flows from holding a stock (dividends and capital gains.) In practice, ten-year bonds are used because bonds with a shorter term than ten years do not reflect the opportunity cost of holding equity investments in which cash flow returns occur over a relatively long time period. The yield on U.S. treasury bonds with different maturities is shown on the graph below. The graph demonstrates that until the financial crisis in 2008, there has been little difference between the 30-year spread and the 10-year spread.

[7] Koller, T., Goedhart, M., Wessells, D., 2005, Valuation Measuring and Managing the Value of Companies, Hoboken, New Jersey, John Wiley & Sons p. 297.
[8] Armitage, S., 2005, p. 87.
[9] T. Copeland, T. Koller and J. Murrin, *Valuation: Measuring and Managing the Value of Companies*, John Wiley and Sons, Inc. 2000.



The duration of stocks with different payout ratios (measured in years) and bonds with different maturities is shown in table below. The table demonstrates that high dividend paying stocks – with a payout ratio of 80% -- are similar to bonds with a term of 25 years.[10]

Bonds		Stocks	
Term (Years)	Duration (Years)	Dividend Payout	Duration (Years)
10	8.11	10%	161.33
15	10.90	20%	106.78
20	13.09	40%	38.89
25	14.80	60%	21.15
30	16.14	80%	14.47
40	18.02	100%	11.00

While application of a long-term bond rate for the risk free rate is a common practice in applying the CAPM, even this seemingly straightforward factor raises a number of practical issues. First, long-term bonds are not at all risk free in terms of real monetary value. If you buy a ten-year bond at an interest rate of 3% and the inflation rate turns out to be 6%, you have lost a lot of money in real terms. This inflation risk is generally more important for bonds than for equities because equity cash flows adjust to different rates of inflation as prices charged for products change. Second, consider a country with changing future expectations of inflation over time – say the expected rate is 15% in the first couple of years and then moderates to 8%. In this case one should use a risk free rate over time consistent with the pattern of cash flows in the model rather than using a constant risk free rate. Third, when investors buy bonds they must pay tax on the interest while if equity investments are made, the tax rate on capital gains and dividends is lower (at least in the U.S.) All else equal, the tax advantage of holding interest rather than dividends should be included in model which may mean the pre-tax cost of equity is less than the cost of debt and the cost of debt should be adjusted for the differential tax treatment. Fourth, if the risk free rate is applied for a country which has a default premium, then one is not really measuring a risk free security but rather the default risk of the country. If the equity investment is not highly related to the country in which it is made – for example an LNG train may sell gas around the world and have little to do with economic conditions in a country – the risk premium can overstate actual risks associated with the investment. In terms of an equation, these factors could be expressed in the following formula which begins with the 10-year Treasury bond that is typically used in the CAPM.

Risk Free Rate =

[10] Mechanics of the duration calculations are contained in the duration exercise on the CD.

Estimation of Beta (β)

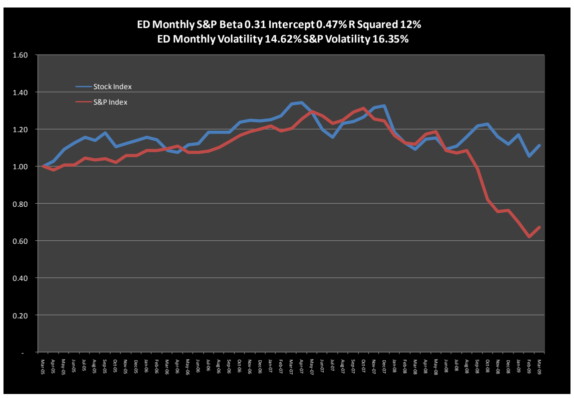
The only factor in the CAPM that measures risk associated with cash flow variation for a particular investment is the beta; all the other factors have nothing to do with the risk individual investments or individual firms. While making a regression analysis to compute beta or finding published data for beta are very simple, there are a variety of practical issues associated with measuring beta. These issues are separated into four different parts below. The first section reviews the definition of beta on a conceptual and statistical basis. Next, betas that are published from different sources which use different time periods and other statistical adjustments are contrasted with betas computed from raw data on stock market returns. Third, numerous statistical questions about the ability of betas to predict relative movements in stock price are considered in light of the financial crisis. Finally, the remarkable way in which large investment analysis firms make an arcane adjustment to beta for mean reversion is analyzed.

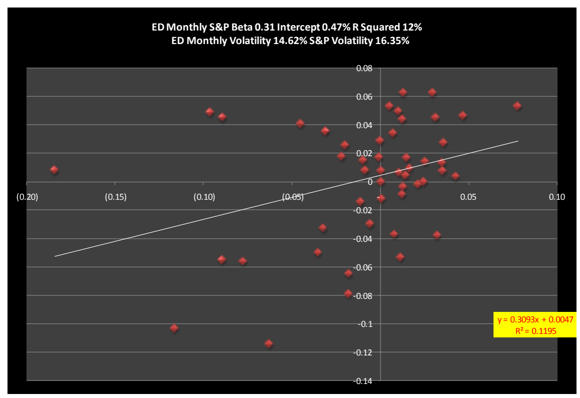
From a statistical standpoint, beta is defined as the slope of a line that measures the relation between returns on a stock and returns on the overall market. More technically, it is normally computed with regression analysis as the covariance between returns on a security and returns on the market divided by the variance in overall market returns. The reason beta measures risk is that any variation in the returns on a stock that are not related to movement in the overall market can in theory be eliminated or diversified away through holding portfolio of many different investments. As in many statistical analyses, the idea behind estimating beta is that there is supposed to be a true underlying relationship that can be discovered through evaluating the historic data. In the case of beta, one assumes that there is some true relationship between the rate of return on a stock and the rate of return on the market, and the statistical analysis is used try to find this true relationship from historic data. In practice, estimation of beta involves evaluation of historic stock price data for companies that are publicly traded and selection of companies that appropriately reflect the risk of the investment being analyzed. This means that in determining the cost of capital for the specific investment, one should try to find companies which have comparable risks to the particular investment in question. In many cases, there are very few publicly listed companies that have portfolios of assets similar to the investment in question. For example, for an airline company studying the risks of investing in an A-380 airbus jet, there are no airline companies that only own that type of plane and there are surely no companies that operate on exactly the same routes as the planned investment. One must instead use betas for a set of airline companies and perhaps adjust them for leverage (it is common to use betas for a group of similar companies, then remove the effect of leverage from each beta and then average all of the asset betas.) The real risk of the investment in a particular investment however will likely be different from the portfolios of planes of the comparison companies.

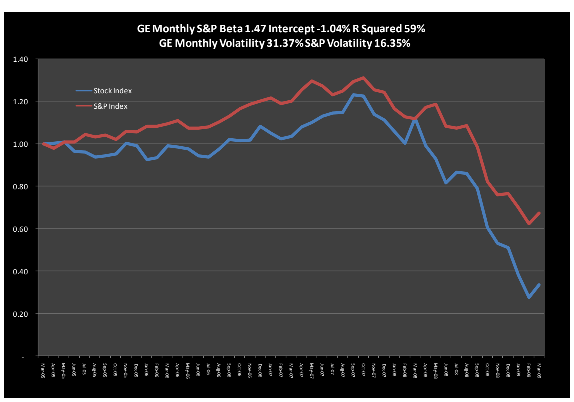
If the true beta is 1.0, then the covariance between the individual security and the market is the same as the variance in market returns and the security is defined to have the same risk as the overall market even if the volatility of the stock is much higher or lower than the volatility of the overall market. Through holding a whole lot of stocks with a beta of one, the portfolio should result in the same return as the market implying that a stock with a beta of one should earn the risk free rate plus the overall expected market risk premium on the whole market. When beta is above 1.0, the return on the stock moves by more than the market and the CAPM implies the cost of capital for the stock is higher than the expected market return ($R_f + R_m$). When the true beta is below 1.0, the cost of capital for the stock is lower than the market return. Derivation of the relationship between beta and cost of capital comes from the fact that risk is defined as the standard deviation of returns on a firm's stock. However notwithstanding the fact that the theory behind the CAPM and the associated statistics resulted in a Nobel Prizes, it is doubtful that most investors really believe that beta captures all of a firm's risks that are relevant to its valuation. If you were to ask people sitting at a bar to give you a definition of risk, few would give you an answer anything like beta and the covariance in returns divided by the variance in the market return. A better definition of risk, especially after the financial crisis, is the simple question of what will happen to an investment when things in the world fall apart; for example, when something like the financial crisis

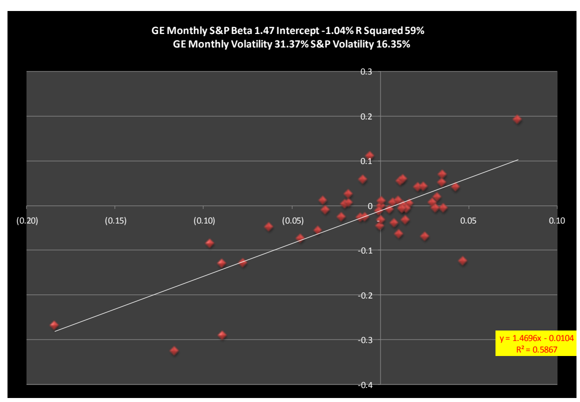
occurs. After the financial crisis, it is clear that one cannot boil risk to a simple mathematical formula or a single statistic.

After a set of publicly listed companies has been selected for comparison, the mechanics of finding the beta is an easy task – much easier than it used to be. You can either directly extract the data from websites such as Finance.Yahoo!.com within seconds, or spend a few minutes calculating beta by extracting stock price data from the same websites. In computing beta yourself, it is common to use five years of monthly returns where the rate of return on a stock is regressed against the rate of return on a wide stock index such as the S&P 500. You can go to Finance.Yahoo!.com and download monthly, weekly or daily stock indices (adjusted for stock splits and dividends) and the overall market to and excel file and then compute the rate of return. (The rate of return should in theory be computed on a continual rather than a discrete basis by using the LN function – LN[current price/last month price].) Once these returns are computed for the stock in question and for the index such as the S&P 500, the beta can be computed by simply using the slope function – SLOPE(stock returns, S&P 500 returns). To make the process quick and easy, the MATCH function can be used to find the common row number between the stock return and the market return. Then the INDEX function can be used to put the returns side by side. Scatter graphs and index graphs that present these calculations for Consolidated Edison (ED) and General Electric (GE) are shown below. An exercise that explains precisely how make the calculations and create the graphs is included on the accompanying CD along with a template model that allows you to easily extract data and calculate statistics.









The beta estimates shown above are different than the estimates made by Value Line and other services that publish beta. For Consolidated Edison, the beta computed from applying the method discussed above results in a statistic of .31 while the Value Line statistic published at the same time was more than double, .65. In the case of General Electric, the computed beta is 1.47 while the Value Line

beta was .95 at the date the graph was created. Thus, even though the statistical analysis is as simple as any regression equation can be – there are only two data series, one coefficient to estimate and no statistical adjustments to make – alternative methods in applying the statistical equations can result large differences in measured beta. One reason for the differences in beta is that some services apply the statistical analysis to weekly data on stock and market returns (e.g. Value Line and Bloomberg) while other use monthly data (e.g. Merrill Lynch)[11]. Another reason for the difference in betas reported by investment services is the length of the historic period used to compute betas. A third reason for the difference in Betas between the betas shown on the graph above and the Value Line beta is that Value Line (and some other services) use a mean reversion adjustment that push betas towards one as shown in the equation below (Merrill Lynch and Bloomberg present both adjusted and unadjusted betas).

$$\text{Beta} = \text{Raw Beta from Statistical Analysis} \times .67 + 1.0 \times .33$$

Statistical ambiguities in computing beta are demonstrated by comparing published betas from different sources. The table below lists the statistics published on the Yahoo, Googlefinance and Value Line websites for a set of regulated utility companies in 2006. The list demonstrates the same companies have betas that are very inconsistent among the different sources. In particular, for the companies with relatively low betas, the Value Line numbers are almost double the betas provided by the other sources – the average is .9 for Value Line versus .49 and .46 from Google and Yahoo respectively. If one would use the Value Line betas instead of other sources and keep other parts of the CAPM the same (i.e. the risk free rate and the market risk premium) one would end up with a very different cost of capital when making investment decisions.

	Beta from Googlefinance Website	Beta from Yahoo Website	Value Line Beta
AGL Resources Inc. (ATG)	0.49	0.13	0.95
Energy East Corp. (EAS)	0.50	0.34	0.90
IdaCorp, Inc. (IDA)	0.80	0.49	1.00
Nisource Inc. (NI)	0.61	0.18	0.90
Peoples Energy Corp. (PGL)	0.37	0.66	0.90
Pepco Holdings Inc. (POM)	0.45	0.57	0.90
SCANA Corp. (SCG)	0.53	0.36	0.80
Southern Co. (SO)	(0.12)	0.23	0.65
WGL Holdings Inc. (WGL)	0.25	0.71	0.80
Wisconsin Energy Corp. (WEC)	0.14	0.46	0.80
Xcel Energy Inc. (XEL)	1.13	0.52	0.90
Median	0.49	0.46	0.90
Average	0.47	0.42	0.86

Differences in beta among companies in the above table arise primarily because Value Line uses weekly data in computing the statistic (as opposed to monthly data) and because Value Line implements a mean reversion adjustment. The mean reversion factor (which is derived from obscure academic articles written in the 1970's) makes a remarkable assumption that betas rapidly converge to 1.0 even though the returns of utility companies such as those listed above virtually never move by more than the returns on the overall market over long periods because returns are protected by regulation. Differences between betas in the above table illustrate that statistical issues clearly have a large effect in the measurement of cost of capital. Before discussing the specific adjustments, it is worth emphasizing that should a couple of very debatable and arbitrary statistical adjustments completely change the estimated cost of capital, then how can the model itself be reasonable or reliable for important investment and policy decisions. Conversely, if there was a true beta and it was really stable over time, and it was similar whether the market returns were very high or very low, all of the statistical debates would fall by the wayside; there would be no need for a mean reversion adjustment; it would

[11] Armitage, Seth, "The Cost of Capital", page 292.

make no difference whether daily, weekly or monthly data were used; and, you would get similar answers if statistical analysis was made over six months or ten years.

A Monte Carlo simulation analysis of beta is included in Appendix 3 which shows that if there is a true underlying beta, there is no theoretical reason for pushing the beta towards a value of 1.0. Given results from the simulation, the first statistical issue addressed is the question of whether it is reasonable to assume betas revert to a mean level of 1.0 over time as is assumed by Value Line and other services. The mean reversion adjustment comes from a study made by a finance professor named Marshal Blume published in 1971 who suggested that betas in the CAPM should be computed using a formula that adds a factor of 1/3 to 2/3 multiplied by the raw beta computed directly from the regression equation. The mean adjustment for betas is often accepted in cost of capital literature for companies because of a supposed tendency of betas to revert to the mean of all companies in the economy and an assumed tendency of high business risk companies to moderate as their business grows into business strategies more representative of a typical company. The mean reversion adjustment has also been attributed to variability in betas over time. If there are sudden jumps in betas followed by falls in the beta, one could smooth the estimates through placing less weight on the recent estimate. For example, if the beta jumps to 3.0 in one period, it is probable that beta for the next period will be lower. Ogier and Rugman explain the adjustment by writing: "there appears to be greater measurement error associated with the betas of extreme values."[12]

Use of a mean reversion adjustment seems to be a relatively minor issue in finance relative to other issues such as whether distributions of returns on stocks follow a normal distribution; whether the CAPM is valid; whether the Black-Scholes model really works; or whether equity cash flows or free cash flows should be used in valuation. However the fact that the mean reversion adjustment continues to be used by well respected firms such as Value Line, Bloomberg and Merrill Lynch is a good example of the general lack of thought and absurdity behind many aspects of financial theory. References are continually made to the Blume article published in the early 1970's even though it is dramatically easier to gather data and evaluate questions than it was at the time. Further, the studies upon which the adjustment is based ignore extremely valuable data regarding the stock price movements during the market crash known as black Monday in 1987, the internet bubble and the financial crisis that resulted from subprime loans and other irresponsible investments.

To evaluate the mean reversion issue and other statistical questions associated with measuring beta, a few companies with betas of above one and regulated utility companies with betas of below one have been selected. The betas for these companies have been computed on the basis of both weekly and monthly data for the entire period over which the stock prices exist. The betas are computed over five year periods for both the weekly and the monthly statistics over to the maximum extent possible depending on the availability of data. Once the five year betas are predicted, the mean reversion adjustment can be tested against a simple forecast using the beta computed for the earlier period. The predicted beta with a mean reversion adjustment is computed using the 1/3 and 2/3 factors discussed above from the beta in the prior period, while the prediction without a mean reversion is simply the beta from the previous period. In terms of equations, the two different predicted betas are:

$$\text{Forecast Beta}_t = \text{Beta}_{t-1}\text{, or}$$

$$\text{Forecast Beta}_t = \text{Beta}_{t-1} \times .67 + .33$$

Once the forecast betas are computed, they can be compared to actual betas to see if the mean reversion produces a better or a worse forecast. The forecast error is computed as:

$$\text{Forecast Error}_t = \text{Actual Beta}_t - \text{Forecast Beta}_t$$

[12] Ogier, T, Rugman, J., Spicer, L., 2004, *The Real Cost of Capital A Business Field Guide to Better Financial Decisions*, Great Britain: FT Prentice Hall, page 54.

As with any statistical measure, its effectiveness should be evaluated by the question of where there is bias in estimating the true value and whether the variance of the error is relatively small. If the forecast error is consistently positive or negative when using mean reversion adjustment or the simple forecast, then there is evidence of bias. From the perspective of efficiency, if the absolute or standard deviation of the forecast is lower for the mean reversion approach, then the mean reversion adjustment can be defined as more efficient.

The table below summarizes results of the mean reversion analysis for a set of companies that have betas of above 1.0 at the end of the sample period. Through simply inspecting trends in betas, it is clear that there is no general tendency for betas to gradually move to 1.0, either if the betas were computed from monthly data or from weekly data. For example in the case of American Airlines, the beta was around 1.4 and then increased over the period when airline companies faced very severe financial pressure after September 11, 2001. The two lines below the monthly and weekly betas for each company named bias report the average of the prediction errors using the mean reversion adjustment and no mean reversion adjustment. If the errors are consistently positive or negative implying bias, then the average will not be close to zero. A test of whether the mean reversion reduces bias in the forecasts is shown underneath the average in the line titled “Smaller Bias.” If the mean reversion formula to compute the subsequent beta produces lower average error in absolute value, the line is labeled MR; if the simple forecast produces lower bias, then the line is labeled is Raw. For each computed series of monthly and weekly betas except case, the mean reversion produces a higher bias than the method which does not use mean reversion. Further, in all the cases, the bias is negative, implying that mean reversion adjustment consistently underestimates the beta for these companies. The deviation in the errors is somewhat lower with mean reversion, meaning when that when beta is very high it tends to come down and when the beta is very low it tends to go up. The problem is that because the average beta is not 1.0, the mean reversion adjustment creates a bias; if the mean reversion adjustment produces somewhat more precise aim, the aim is at the wrong target.

Betas Analysis for Companies with High Betas

			International Paper	Ford	Alcoa Aluminum	Citibank	Catepiller	Americal Airlines
Company Name								
Ticker Symbol			IP	F	AA	C	CAT	AMR
Yahoo Beta			2.57	2.77	2.14	2.98	1.83	1.62
Value Line Beta			1.45	1.55	1.70	2.15	1.30	1.65
Financial Crisis Beta			2.70	1.44	2.88	4.53	1.86	1.66
Monthly Beta	From	To						
	Dec-69	Dec-74	**1.06**				**1.26**	
	Dec-74	Dec-79	**1.27**	**0.95**	**1.05**	**0.72**	**0.94**	
	Dec-79	Dec-84	**1.45**	**0.70**	**0.90**	**0.78**	**1.22**	**1.04**
	Dec-84	Dec-89	**1.21**	**1.24**	**1.11**	**1.28**	**1.19**	**1.42**
	Dec-89	Dec-94	**1.04**	**1.16**	**1.06**	**2.04**	**1.14**	**1.43**
	Dec-94	Dec-99	**1.02**	**1.04**	**1.00**	**1.83**	**0.82**	**1.35**
	Dec-99	Dec-04	**0.94**	**1.28**	**1.62**	**1.30**	**1.06**	**3.61**
	Dec-04	Dec-09	**2.32**	**2.58**	**2.34**	**3.01**	**1.88**	**1.75**
	Bias using Mean Reversion		**(0.23)**	**(0.29)**	**(0.26)**	**(0.49)**	**(0.12)**	**(0.40)**
	Bias using Simple Model		**0.18**	**0.27**	**0.22**	**0.38**	**0.09**	**0.14**
	Smaller bias		**Raw**	**Raw**	**Raw**	**Raw**	**Raw**	**Raw**
	Deviation using Mearn Rev		**0.31**	**0.35**	**0.27**	**0.50**	**0.26**	**0.57**
	Deviation using Simple		**0.33**	**0.36**	**0.26**	**0.54**	**0.29**	**0.66**
Weekly Beta	From	To						
	Dec-69	Jan-75	**0.98**		**0.93**			
	Jan-75	Jan-80	**1.34**	**0.79**	**1.33**	**0.55**	**0.96**	
	Jan-80	Jan-85	**1.18**	**1.03**	**0.99**	**0.89**	**1.04**	**1.36**
	Jan-85	Jan-90	**1.31**	**1.09**	**1.43**	**1.20**	**1.23**	**1.34**
	Jan-90	Dec-94	**1.06**	**1.26**	**1.10**	**1.49**	**0.98**	**1.28**
	Dec-94	Dec-99	**0.58**	**0.96**	**0.62**	**1.92**	**0.79**	**1.35**
	Dec-99	Dec-04	**0.85**	**0.96**	**1.09**	**1.25**	**0.88**	**2.36**
	Dec-04	Dec-09	**1.48**	**2.30**	**2.04**	**2.81**	**1.43**	**2.00**
	Bias using Mean Reversion		**(0.09)**	**(0.26)**	**(0.18)**	**(0.45)**	**(0.07)**	**(0.76)**
	Bias using Simple Model		**0.07**	**0.25**	**0.16**	**0.38**	**0.08**	**(0.24)**
	Smaller bias		**Raw**	**Raw**	**Raw**	**Raw**	**MR**	**Raw**
	Deviation using Mearn Rev		**0.27**	**0.28**	**0.43**	**0.49**	**0.17**	**0.11**
	Deviation using Simple		**0.32**	**0.30**	**0.49**	**0.51**	**0.19**	**0.10**

The next table shows similar statistics for utility companies with low betas. As with the high beta companies, the mean reversion adjustment creates a clear bias as none of the companies have any evidence that the betas are moving towards 1.0. This time the bias is consistently positive meaning that the mean reversion adjustment consistently over-estimates the beta. Further, the absolute deviation is also lower without the mean reversion adjustment implying that the mean reversion adjustment does not add any precision. Betas computed by Value Line that include the mean reversion demonstrate the bias. From 1994 through 2009, the Value Line beta overestimated every single beta computed from actual changes in stock prices relative to overall changes in the market. For companies in the table above with consistent betas below 1.0, if some kind of mean reversion adjustment improves estimation of beta, the reversion should be to the average and not to an arbitrary value of 1.0.

Betas Analays for Utility Companies with Low Betas

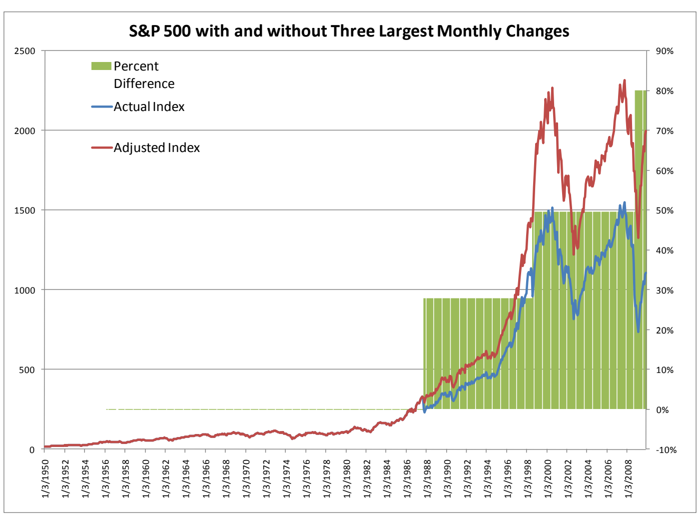
	From	To	WEC	AGL	WGL	NSTAR	ED	XEL
Yahoo Beta			0.36	0.42	0.11	0.22	0.25	0.26
Value Line Beta			0.70	0.75	0.65	0.55	0.65	0.60
Financial Crisis Beta			0.52	0.27	0.06	0.10	0.17	0.32
Monthly Beta	From	To						
	Jan-70	Jan-75					0.67	
	Jan-75	Jan-80					1.05	
	Jan-80	Jan-85					0.13	
	Jan-85	Jan-90	**0.28**	**0.20**	0.34	**0.36**	0.32	**0.44**
	Jan-90	Dec-94	**0.59**	**0.48**	0.38	**0.66**	0.58	**0.36**
	Dec-94	Dec-99	**0.06**	**0.37**	0.27	**0.24**	0.15	**0.33**
	Dec-99	Dec-04	**0.05**	**0.29**	0.21	**0.29**	(0.05)	**0.09**
	Dec-04	Dec-09	**0.38**	**0.38**	0.15	**0.25**	0.29	**0.13**
	Bias using Mean Reversion		**0.22**	**0.18**	**0.28**	**0.23**	**0.25**	**0.31**
	Bias using Simple Model		**0.03**	**0.04**	**(0.05)**	**(0.03)**	**(0.05)**	**(0.08)**
	Smaller bias		**Raw**	**Raw**	**Raw**	**Raw**	**Raw**	**Raw**
	Deviation using Mearn Rev		**0.18**	**0.10**	**0.16**	**0.16**	**0.34**	**0.17**
	Deviation using Simple		**0.14**	**0.08**	**0.04**	**0.12**	**0.39**	**0.06**
Weekly Beta	From	To						
	Jan-70	Jan-75					**0.75**	
	Jan-75	Jan-80					**0.60**	
	Jan-80	Jan-85					**0.52**	
	Jan-85	Jan-90	**0.43**	**0.30**	**0.35**	**0.34**	**0.46**	**0.53**
	Jan-90	Dec-94	**0.46**	**0.43**	**0.34**	**0.63**	**0.70**	**0.62**
	Dec-94	Dec-99	**0.13**	**0.36**	**0.36**	**0.29**	**0.31**	**0.27**
	Dec-99	Dec-04	**0.36**	**0.45**	**0.43**	**0.39**	**0.26**	**0.51**
	Dec-04	Dec-09	**0.51**	**0.70**	**0.64**	**0.56**	**0.50**	**0.53**
	Bias using Mean Reversion		**0.19**	**0.10**	**0.14**	**0.14**	**0.55**	**0.17**
	Bias using Simple Model		**0.02**	**0.10**	**0.07**	**0.06**	**0.17**	**(0.00)**
	Smaller bias		**Raw**	**Raw**	**Raw**	**Raw**	**Raw**	**Raw**
	Deviation using Mearn Rev		**0.12**	**0.08**	**0.08**	**0.10**	**0.58**	**0.10**
	Deviation using Simple		**0.10**	**0.08**	**0.04**	**0.13**	**0.41**	**0.10**

The fact that the mean reversion adjustment to beta continues to be made by well respected financial companies such as Value Line, Merrill Lynch and Bloomberg and that it is almost universally accepted by academics is a symptom of serious problems with the current state of financial analysis. Errors in using mean reversion are confirmed dramatically in the analysis below which evaluates the approach that most accurately predicted betas during the financial crisis which shows that the use of mean reversion aggravates the error rather than reducing the error. It is certainly true that betas are unstable, but making an arbitrary adjustment to move all betas toward 1.0 so that they seem to be more stable makes no sense from either a statistical perspective or from a theoretical standpoint.

A second statistical estimation question regarding beta is whether it somehow better to use daily rather than weekly or monthly betas. Seth Armitage reports that use of weekly data rather than monthly data tends to produce higher betas for low beta stocks while the reverse is true for high beta stocks, namely that weekly data produces lower estimates of beta than monthly beta. A remarkable thing about the differences in observed betas for different time periods is that there is no explanation for why a bias would exist and if the bias does exist, which time frame results in the best estimate. If the difference in estimated betas makes an important difference in the estimated cost of capital which can affect investment decisions and one does not know why there are differences, then how can the whole CAPM model possibly be useful in practical decisions with respect to making an investment. To investigate this issue, the average of monthly and weekly betas before the financial crisis is presented for the same companies used in the analysis above. The average of monthly betas is indeed somewhat lower than weekly betas for most companies that have relatively high betas and the reverse is true for companies with relatively low betas. From this very limited sample it appears that the differences between monthly and weekly betas are not simply an old wives tale.

Betas Analysis for Companies with High Betas						
Company Name	International Paper	Ford	Alcoa Aluminum	Citibank	Catepiller	Americal Airlines
Ticker Symbol	IP	F	AA	C	CAT	AMR
Monthly Average Beta	1.14	1.06	1.13	1.33	1.09	1.77
Weekly Average Beta	1.04	1.01	1.07	1.22	0.98	1.54
Pre- Financial Crisis Monthly	0.94	1.28	1.62	1.30	1.06	3.61
Pre-Financial Crisis Weekly	0.85	0.96	1.09	1.25	0.88	2.36
Financial Crisis Beta	2.70	1.44	2.88	4.53	1.86	1.66
	WEC	AGL	WGL	NSTAR	ED	XEL
Monthly Average Beta	0.25	0.34	0.30	0.39	0.25	0.30
Weekly Average Beta	0.34	0.39	0.37	0.41	0.43	0.49
Pre- Financial Crisis Monthly	0.05	0.29	0.21	0.29	(0.05)	0.09
Pre-Financial Crisis Weekly	0.34	0.45	0.43	0.39	0.26	0.51
Financial Crisis Beta	0.52	0.27	0.06	0.10	0.17	0.32

Given all of these statistical issues, the question is still which method should be used when applying the CAPM. The ultimate test is how good of a job the various statistical measures do of predicting the percent changes in stock prices that occurred during three major market movements during the past quarter of a century. As stated above, these large market movements created most of the money that was lost or made during the past thirty years. The graph below shows actual movements in the S&P 500 index compared to an artificial index in which the three largest percentage monthly changes (out of 719 months since 1950) are replaced with zero.[13] The graph demonstrates that without these few large changes, the S&P index would be almost 80% higher than the actual index. This confirms that the important data points for investors are the large market changes rather than the minor movements. As a corollary, the important beta is the beta that occurs when the market changes are very large. If the beta was estimated to be 1.0 from historic data analysis, but the stock moved down by 100% when the market fell by 25%, then the beta that really matters is 4.0, and the estimated beta of 1.0 from historic data did a poor job of measuring risk. Similarly, if the stock moved up when the market crashed, then the true beta that matters is negative and if the beta estimated from historic data was positive, it is not very relevant to the real risk of the stock.



[13] This is similar to a graph presented by Nicholas Taleb in "Black Swan." The graph is created by using the MATCH, INDEX and LARGE functions in excel.

Through focusing on the beta from the financial crisis, it is clear that betas from estimated historic data are poor estimators of the period of extreme market decline from August 2008 to February 2009. The financial crisis beta is simply the percent change in the stock price over the period divided by the percent change in the S&P 500 over the same period. For example, the stock price of Citibank was 18.7 at the beginning of August 2008 and it had fallen to 1.5 in February 2009 implying a percent change (using the LN function) of 252%. The similar percent change in the S&P 500 was 55.6% over the same period, meaning that the actual beta was 252%/55% or 4.53. Investors whose share price fell from 18.7 to 1.5 cared about the change that occurred when most of their value dissipated, not other periods. In terms of predicting the beta during the financial crisis, none of the methods performed very well. The table above demonstrates this where one can insptect the pre-crisis beta relative to the actual beta, it is apparent that neither the weekly nor the monthly statistic were anywhere close.

In addition to the questions addressed above, an issue involving what market index to address for non-U.S. companies sometimes arises. For example, if a stock is traded on the Cairo stock exchange in Egypt, should the Cairo exchange be used in the regression analysis or is it better to use a relatively wide index such as the S&P 500. Similar issues come up with respect to whether the market index should include assets other than stocks such as real estate and bonds. Given all of the other problems with the CAPM, attempting to find a more refined index with other assets is not a very big concern. However, use of small regional exchanges does pose an obvious problem. In computing the beta of Nokia, for example, if the Helsinki stock exchange is used, the beta will obviously be close to one as Nokia represents a large percent of the overall market capitalization of the entire exchange. Similar issues arise for HSBC in Hong Kong and financial stocks for the Swiss stock market. Yet despite this obvious problem and the fact that investors are able to diversify all over the world, well known and well respected investment services compute betas using the regional indices.

Estimation of the Expected Market Risk Premium (R_m)

The above sections demonstrated that a number of issues arise with respect to the risk free rate and beta; these issues alone are probably enough to render the model useless from a practical perspective. Ambiguities caused by the beta and market risk premium however are relatively minor relative to the third factor in the model -- the level of the return investors expect to earn over and above the risk free rate in order to take the risk of general equity investments rather than risk free investments (the expected market risk premium or EMRP or R_m.) The EMRP is a very important number in financial economics which affects things ranging from the amount of money that should be put into pension fund reserves to the debate about privatizing social security. The whole basis of the CAPM is that once the number is known, we can then dial that market return upwards or downwards depending on how risk of the particular investment relates to the risk of equity investments in general. The problem with EMRP is that although it is intensely studied, it is virtually impossible to compute. The number changes with perception of risk; it cannot reasonably be computed from historic data and it cannot realistically be computed from the applying the dividend discount model to a wide portfolio of stocks. In the end, one simply cannot avoid the fact that the market risk premium is a psychological concept.

To address the issue, this section separates the difficult question of estimating the EMRP into the following issues:

- What is the theory underlying the Expected Market Risk premium and what general level of results should be expected;

- Is it reasonable to estimate expected returns from measuring the level of achieved returns over long time periods;

- Can the estimated market risk premium be estimated from application of the dividend discount model to a large number of stocks in the market; and,

- What is the risk premium applied by financial professionals and is this estimate useful in making investment decisions.

Before delving into possible ways to estimate what investors in equity securities require to compensate for the additional risk associated with equities over risk free bonds, it is useful to review a few general theoretical aspects of the number. The R_m has been the subject of a very large body of research – as mentioned above it has been called the most studied number in all of finance. Some of the points that can be derived from the research include:

- First, the R_m is expressed in real terms and is not a nominal number because changes in inflation are subtracted in the risk free rate. This means the R_m does not necessarily increase or decrease when interest rates change, when the expected rate of inflation changes or when the stock market fluctuates. It would be wrong, for example, to update this number because of changes in the interest rate.

- Second, the R_m comes from the general risk preferences of agents in an economy for equities relative to risk free securities. If people did not have risk aversion for equities relative to risk free bonds, the R_m would be zero. This means changes in the R_m are driven by changes in general preferences for equities relative to risk free securities. Because the R_m comes from risk preferences, it only changes as the general attitudes to risk change.

- Third, the R_m is an expected number, rather than a number that can be directly measured from actual realized returns. The fact that the R_m is an expected number means that it does not change when the stock market goes up or down because realized earnings change or because general economic activity is robust or depressed.

- Fourth, the R_m is an economy wide number not unique one particular company. Unlike beta, which measures the risk associated with a specific company, the R_m is the same whether it is used in valuing a paper company, gauging the rate of return for an oil project or assessing the share price of an airline company.

- Fifth, R_m estimates presented in some text books of 7-8% or by cost of capital experts in litigation cases of above 10% are not plausible from a macro equilibrium perspective. To see this, assume that a country has a stable population and that because of capital and labor productivity the overall economy will grow in real terms by 2% per year. In this case, should investors as a group – including investors in risk free government bonds, risky corporate bonds, real estate and other investments as well as risky equities – make a return of more than 2% per year, then somebody else in the economy, i.e. labor – will be worse off. For example, if equity investors earned a risk free return of 1% plus a premium of 8% for a total real return of 9%, then their income would grow much faster than the overall economic growth rate of 2%. This growth rate would have to come from non-equity holding segments of the economy that would have to have their incomes grow by a rate lower than the overall growth rate of 2%. While the income of investors can surely grow at a faster rate than the overall economy for short periods of time, in the long-run these assumptions cannot be stable and would cause extreme dispersions in income between equity holders and other agents in the economy.

- Sixth, the market risk premium numbers have been hotly debated for the decades as data on markets other than the U.S. and longer or shorter periods suggest lower returns than those typically used earlier. In one of the articles which evaluated implicated cost of capital expectations from decomposing P/E ratios, Richard Grinold and Kenneth Kroner conclude: "…examining the decomposition of expected returns makes it apparent that such an optimistic overall view [a 5.5% equity premium] requires one to be either optimistic about each component of the expected equity return or irrationally optimistic about at least one of the components."[14] In a second article, Jannathan and Meier note that "recent research

[14] Grinold, Richard and Kroner, Kenneth, "The Equity Risk Premium: Analyzing the long-run prospects for the Stock Market" InvestmentInsights, 2002, page 152.

indicates that the average market premium might have been as low as half of the market premium during the last two decades."[15] The authors also conclude that the true equity premium has been in the range of 1% to 3%. In another oft cited paper published in 2001 by Claus and Thomas titled "Equity Premia as Low as Three Percent? Evidence from Analysts' Earnings Forecasts for Domestic and International Stock Markets," the authors conclude that "for each year between 1985 and 1998, we find that the equity premium is around three percent (or less) in the United States and five other markets."[16]

When the CAPM was initially developed, it was common to estimate the EMRP through comparing actual historical stock market returns with the risk free rate. While this approach of using historic return data (which often used published estimates by a consulting firm named Ibbotson and Associates) was often used for U.S. stocks from 1929 producing a risk premium that ranges between 6-8%, it is wrong from both a practical and theoretical standpoint. Before considering these flaws, it is instructive to understand why historic returns were used in the first place. Pretend you lived in a world where there is a long history of equity and risk free returns and the risk premium remains constant because of the psychology of people's attitudes to risk and because of the stability of real growth in the economy. Further, make the heroic assumption that over the long-term, actual returns earned on stocks somehow approximate those that were initially expected. In this hypothetical world, if returns on stocks fall below the minimum expected return, then stock prices will decline and realized returns on equity will increase for new purchasers of stock. On the other hand, if returns on stocks produce returns above those that are expected, then stock prices will rise and the realized returns on equity will fall for new investors. Over a very long period, the movements in prices should balance out and such that the actual return will approximate the expected return.

The problem with this idea of using long historical periods of stock prices and risk free rates is that actual returns from the last fifty or even one hundred years may not really reflect the expected returns which investors needed at the time to be compensated for risk. The actual realized returns are but only one path out of an infinite number of possible realizations that may have been expected by investors – investors in 1900 did not expect two world wars, the great depression, the advent of television etc. A general conclusion is that the size of the actual equity market premium realized in the market has been higher than prospective expected equity returns. For example, Seth Armitage summarizes the current research: "Almost all researchers on this question agree that the premium in the twentieth century in the United States … has turned out to be larger than investors expected it to be."[17] Further, even if one could somehow measure the rate of return investors expected to earn above the risk free rate, historical expectation in 1900 may not be a good proxy for future expectations in 1930. If investors became more risk averse in 1930, then the required return would increase and stock prices would be bid down. This decline in stock prices would be observed as a lower and not a higher required premium when averaging long periods of data. In addition to changes in risk perception, there are changes in the supply of equity capital, changes in technology and government policy that may lead to expected equity returns differing from expected returns from historic periods. For example, if productivity increases in the economy, the expected return should increase for all providers of capital as discussed above. From a practical perspective there are many data questions about whether measured returns from history have an upward bias because of the bankruptcies of companies that once were part of the overall market but are not included in returns. The final nail in the coffin of using historic data should be the financial crisis. When the stock market lost half of its value, measures of the EMRP using historic data will obviously decline as risk perceptions have increased.

A second approach to measurement of the market risk premium is attempting to derive the cost of capital for each company in the whole economy through using the expected earning or dividend growth model introduced above. With extensive published data on estimated earnings growth, stock prices,

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[15] See Jannathan , Ravi and Meier, Iwan , *Do We Need the CAPM for Capital Budgeting,* September 25, 2001, Kellog School of Management

[16] Claus, J and Thomas, J ,2001, "Equity Premia as Low as Three Percent? Evidence from Analysts' Earnings Forecasts for Domestic and International Stock Markets", Journal of Finance, 55, No 5., pp. 1629-1666.

[17] Armitage, S., 2005, *The Cost of Capital Intermediate Theory*, Cambridge University Press, p. 98.

dividends and earnings one can fairly easily compute the cost of capital for each company in the S&P 500. As the most difficult part of the CAPM, namely the EMRP, then is derived from the dividend discount model or the P/E equation, the CAPM then boils down to the dividend discount model. An example of attempting to measure current market expectations is a well known study by Fama and French, published in 2002, that uses the dividend discount model and finds that "estimates [of the equity market risk premium] for 1951 to 2000, 2.55 percent and 4.32 percent, are much lower than the equity premium produced by the average stock return, 7.43 percent."[18] If the CAPM is derived from the dividend discount model, then all of the problems with the dividend discount model involving impossible to estimate expected growth rates and optimism biases from sell side analysts discussed in Appendix 2 also become problems with the CAPM. Other problems with applying the dividend discount model are that the it is fragile for companies with high growth and/or low dividend payout ratios and that the five-year growth forecasts made by sell side which are generally used are all but meaningless. Appendix 2 shows that with slightly different assumptions, the model produces a wide range of different cost of capital estimates for the same company.

A third and probably best method for estimating EMRP is to survey market participants or investment bankers and ask them what they require. The idea of looking to various sources to see what others use seems reasonable even if one does not know what the true number is. Given the difficulty in coming with an EMRP, if we knew what others use, then we would at least be consistent with the way in which others value investments and the valuation of a particular investment relative to other would be consistent. Sources of data on the market risk premium and the overall cost of capital include prospectuses from mergers, academic articles[19], surveys and pension plan documents. There are a few problems with using surveys from other people. First, if surveys are used, then you are simply using the subjective opinions of others; who is to say that the opinion of the CFO a large company or the opinion of an analyst at Lehman Brothers is better than the opinion of your mother. The market perception of risk is not driven by experts with MBA degrees from top business schools. More importantly, if the opinions of so-called experts are used to determine the EMRP, then the whole CAPM simply boils down opinions of others. Instead of using the CAPM for individual stocks, one could simply ask investors what kind of return they require and have the same statistical precision. It would be hypocritical to criticize other methods for using subjective judgment.

Given the volume of research and the continuing debate surrounding R_m, the real answer is that we must admit that we do not know what the real number is and it cannot be defined in an objective analytical manner. Furthermore, it is probable that the risk premium changes by a lot as people who make up markets have different psychological attitudes towards risk. This problem has an implication that people who believe finance can be defined by mathematics is difficult to accept – namely that the whole CAPM which is behind so much finance theory boils down to psychological beliefs about risk and the market.

Risk Neutral Valuation and the Arbitrage Pricing Model

Given all of problems with the CAPM ranging from statistical problems in estimating beta to the impossibility of estimating the EMRP, it is certainly understandable to look for a better model. After development of CAPM, one attempt at improving CAPM was a model named the arbitrage pricing model ("APM".) The APM adds factors other than the risk premium to the regression of stock returns which can include the rate of inflation, changes in default rates, movements in the term structure of interest rates, industrial production and other economic variables.[20] When the APM is presented after the CAPM in many finance texts, it generally looks like a statistical hodgepodge that does not have any real predictive value. To practically implement the APM, multiple betas must be estimated and the expected rates of change for all of the variables must be estimated such as industrial production must be

[18] Fama, E. and French, K., 2002, "The Equity Premium", Journal of Finance, Vol. 57, No. 2, pp. 637-659.
[19] Damodaran, Aswath "Equity Risk Premiums (ERP): Determinants, Estimation and Implications - A post-crisis Update", October 2009, adamodar@stern.nyu.edu
[20] Damarodan, page 51.

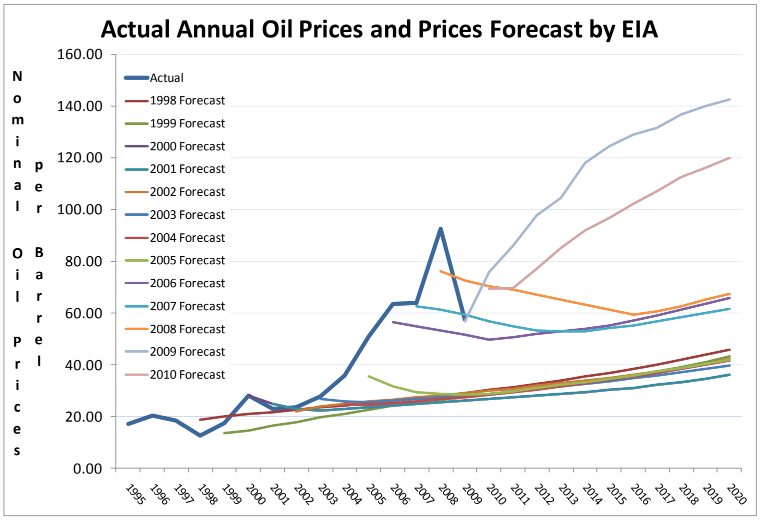
predicted. The model seems to only compound problems with estimating the EMRP discussed above. Further, the model seems to be able to come up with some more observed statistical accuracy by mining data, but there is not much intuition to adding factors such as expected inflation rates in the model. Since the model was first proposed in the mid-1980's it has not had much traction in terms of practical application as it is more difficult to estimate the multiple betas and to come up with expected premiums for each of the factors. In sum, given the added data and statistical complexity, the model does not seem to be the answer to problems with the CAPM.

Despite the seeming uselessness of the APM model, the general idea of finding variables that can be used in arbitrage strategies can be quite useful for valuation in limited circumstances. In some cases the tasks of estimating cash flow and then adjusting the discount rate can be dealt with by creating a hypothetical arbitrage strategy if it is possible to hedge cash flow. Rather than discussing the model in terms of fancy regression models, the potential advantage of the model can be explained in the context of a simplified example. With a simple example, the general idea of adding potential for arbitrage can provide some interesting answers to cost of capital, risk and value issues. The benefits from the APM do not come from simply throwing a few variables into a regression equation; instead, the APM shows that forecasts in financial models should be made from forward prices and cost of capital should be close to the risk free rate.

To see how the APM can be practical, consider the investment involving the purchase of an exiting oil field in Texas. As with any investment, the value of the oil field depends on the cash flow forecast and the risk associated with that forecast. Assume that the operating costs of the oil field are small and are fixed. Further, pretend that the amount of production provided in the reserve reports is known with certainty on a year by year basis. In this case the only remaining risk that can cause any variation in future cash flow is volatility in the price of oil. Valuation of the oil field seems to require two things that are very difficult to forecast: the first is a forecast of the price of oil and the second is a discount rate applied to forecast should reflect the risk that the oil price forecast will be wrong.

In thinking about the discount rate to apply to the expected cash flow of this investment, you could begin with the fact that the volatility of the price of oil is about twice the volatility of the S&P500. This would seem to suggest, at first blush, that the discount rate applied in valuation should be quite high. You could get more sophisticated and even measure the beta of oil and inspect the beta of independent oil companies. With the beta, you could then compute the cost of capital using the CAPM (although many would shake their heads if one suggests that oil investments have less risk than other investments because changes in the price of oil are not that closely correlated with changes in the market). The beta for independent oil and gas companies such as Devon Energy discussed below is about 1.0, while the beta for major integrated oil companies such as ExxonMobil is about .60. Of course, these cost of capital estimates would be subject to all of the ambiguities regarding statistical adjustments to beta and impossibility of measuring the EMRP.

With an estimated discount rate for the oil investment in Texas, you would still have to come up with a forecast of the price of oil to forecast cash flow. Perhaps you would make some sophisticated analysis of the long-run marginal cost as discussed in chapter 6; alternatively you could use the forecast made by a sophisticated forecasting institution such as the U.S. Energy Information Agency (EIA); or, you could hire a consulting company to make forecasts. Any of these methods should give you an uneasy feeling as shown in the graph below which compares actual oil price forecasts with those made by the EIA. Errors in the oil price forecast relative to actual prices and variations in forecasts from one year to the next are a good illustration of relying on independent experts when making forecasts; the dramatic and consistent errors demonstrates that relying on any forecast is close to useless.



In thinking a bit more about the valuation problem for the oil field, we could come up with an entirely different approach using arbitrage concepts. We could use forward contracts traded on the NYMEX to fix the future price of oil associated with the future production (which is assumed to be know). If prices were hedged, there would be no more cash volatility at all and therefore the cash flows should be valued at the risk free rate. We do not have to make a forecast of the price of oil and we do not have to make an estimate of the cost of capital, whether through a CAPM analysis or some other method. It would be difficult to make an argument that the valuation established from forward markets is somehow less accurate than the forecast that has both the speculative oil price forecast and the impossible to estimate cost of capital. The use of forward prices combined with a risk free rate could be also termed risk neutral valuation.

The crucial point about valuing the project through locking in cash flows and using the risk free rate is that we do not actually have to go to the NYMEX and lock into contracts. The oil field has exactly the same physical characteristics whether forward contracts are used or whether spot price exposure is taken – the costs of operating the oil field are the same and the amount of production is the same and so forth. Whether or not contracts are actually used, the prices from forward contracts can be applied in cash flow modeling in order to make a valuation of the oil field. In this example there is potential for arbitrage that can be used in valuation and we could call the process an arbitrage pricing model.

The idea of using hedging contracts and arbitrage in valuation analysis can include debt as well as equity investments. If we are lending money to the oil project and there were no hedging of contracts, a lender would like to know what kind of credit spread that should be added to the risk free rate in pricing a loan (or alternatively finding the cost of capital for debt). To do this, one could find the break-even price of the loan before which the loan defaults as described in the last chapter. For example, perhaps the oil price has to be above $30/barrel for the loan to be repaid in full without default. Since the loan loses money when the oil price falls below $30/barrel, a lender could purchase a put option with a strike price of $30/barrel which would offset the loss from lending to the oil field. The cost of buying the option – the premium on the option – could then be compared to the profit made on the loan from earning the credit spread. If the cost of the option is more than the credit spread earned on the loan, then the credit spread is not high enough and the loan should not be made – if you want to take a risk on the price of oil falling, you could sell a put option and make more money. On the other hand, if the cost of the option is less than the value of the credit spread on the loan, the lender could make a profit without taking risk through buying put options and making the loan. This example demonstrates that as with the cost of equity, the cost of debt can be established using arbitrage pricing concepts.

Real world valuations are virtually never as simple as the oil field investment discussed above. However, if a stock is affected by the price of oil, the price of corn, the price of copper, the price of

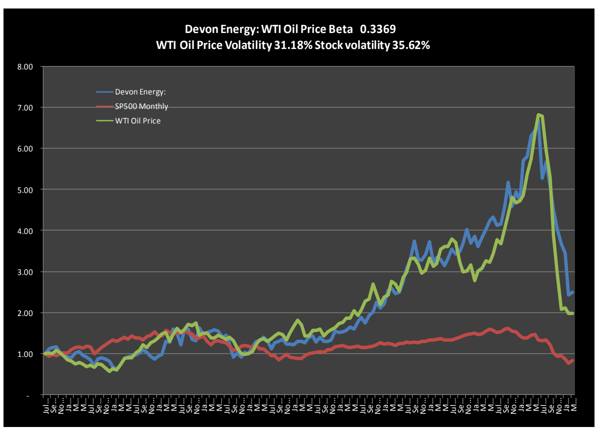
natural gas, the interest rate or other factors that can be hedged, the cost of capital estimate can be adjusted to account for the possibility of hedging in an analogous manner to the simple oil field investment discussed above. In the extreme, if all of the volatility in a stock price can be removed through buying different futures contracts, then the relevant volatility for computing the cost of capital is the remaining volatility after hedging, which would be zero. For other cases where some but not all of the volatility can be hedged, the following three step process could be used to evaluate how much the cost of capital should be reduced relative to the cost of capital computed from a simple CAPM because of the possibility of hedging:

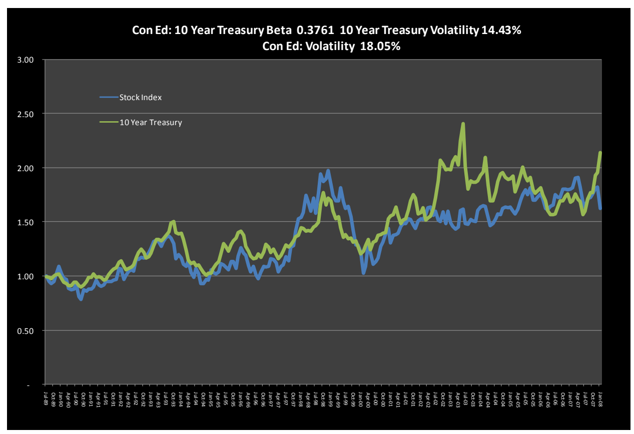
Step 1: Create a regression model of the stock price and the variable or variables that can be used to hedge the volatility of cash flows (this should use forward market data and appropriate autocorrelation adjustments.)[21]

Step 2: Compute the change in the residual from period to period divided by the prior period stock price to determine the volatility that cannot be hedged.

Step 3: Regress the percent change in the residual from step 2 against the change in the overall market to compute the adjusted beta.

To demonstrate how arbitrage concepts can be used in practice, examples of Devon Energy, an oil company whose value is sensitive to oil and gas prices and Consolidated Edison, a utility company sensitive to interest rates are presented below. The graphs show that the Devon stock price is closely related to the price of oil and that the Consolidated Edison stock price closely follows the interest rate. Before accounting for potential hedging, the beta of Devon is .79 and the beta of Consolidated Edison is .31. After using the three step process to account for possible arbitrage and hedging, the beta of Devon Energy falls to .55 and the beta of Consolidated Edison falls to .12.





Reasonable application of the arbitrage pricing model is not simple, because one must make a time series model and collect data on variables that can be used to hedge movements in a stock price. While implementation is difficult, there are a couple of practical implications from this analysis. First, the analysis casts even more doubt around the CAPM. Second, when developing valuation models in industries where hedging is possible, it is better to use forward market data in financial models and use relatively low cost of capital estimates.

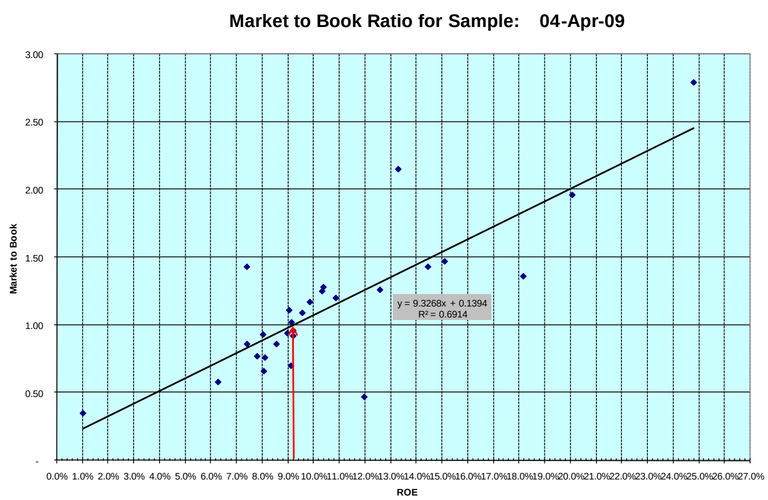
Computing Cost of Capital from Market to Book Ratios

[21] A regression process that adjusts for autocorrelation is included on the CD.

The underlying problem with the CAPM, the dividend discount model and the APM is that the cost of capital is not an observable number. This section presents a way of finding the cost of capital that is not taught in business schools which directly evaluates whether a company is earning more or less then its cost of capital. The alternative model uses the fact that one of the few things that can be demonstrated in working with financial ratios is that when the market to book ratio is equal to one, the earned return on equity is equal to the cost of equity. A proof of the fact that invested capital equals market capital when the return on invested capital equals the cost of capital is included in Appendix 1. If the return exceeds the cost of capital, then the market value exceeds the cost of capital and if the return is below the cost of capital than the market value of capital is below the invested capital. Given this relationship one can generally observe a strong positive relationship between the expected earned return on invested equity capital (ROE) and the ratio of the market value of capital to the invested equity capital (M/B Ratio). The level of the ROE at which the market to book ratio is one can be used to define the cost of capital as long as the ROE is reasonably constant and the equity investment reported on the balance sheet represents the amount of money invested in the company.

To apply the market to book approach, one should select a sample of companies with reasonably similar risks and stable rates of return (if companies have negative returns or very high returns they should be removed from the sample.) Using this sample you can evaluate the range in returns in an industry along the distribution of market to book ratios and develop a statistical analysis that derives the return on equity consistent with a market to book ratio of one. To find cost of capital, one can create a simple regression equation that predicts the ROE as a function of the M/B using the sample data. The cost of capital can then be derived by plugging a value of one into the regression formula as demonstrated in the equation and the graph below where the estimated cost of capital is about 9.0%.[22]

ROE = a + b x M/B Ratio, Cost of Capital = a + b x 1.0



Today's technology makes it very easy to compute relatively complex quarterly three stage dividend discount models as described in Appendix 1, or to calculate alternative betas with different types of statistical techniques from historic data. Coming up with effective ways to see if your results are reasonable requires more important skills, including the ability to admit that model calculations can be wrong. Use of the M/B ratio as described in the above equation to establish or at least check the cost of capital is an example of using a relatively simple back of the envelope model to verify the results of more complex analysis (the importance of this type of simple analysis was introduced in chapter 2). Recall that use of the word simple, does not mean the process is easy; rather the checks should reflect financial experience and business judgment. When simple analyses such as the M/B equation deviate

[22] The file named financial ratio download.xls included on the CD allows you to automatically compute the cost of capital from market to book ratios.

from the results of more detailed models, one should check the inputs and logic of the complex models rather than asserting that a check using simple relationships is not sophisticated enough.

Applying an M/B equation to verify other cost of capital models has a big advantage in that the analysis does not require subjective estimates of indefinite cash flow growth rates or the expected market risk premium that are virtually impossible to measure on an objective basis. Using techniques such as the market to book ratio can sometimes provide a test of blind conformity that can distort measurement of cost of capital and expected cash flow. Some numbers for expected earnings growth or the market risk premium can take on a life of their own in financial markets, despite clear contradictions with common sense and basic financial principles. High expected cash flow growth projected by sell side analysts is analogous to the belief that U.S. housing prices could never fall before the financial crisis. This belief persisted even though prices had doubled in a few years and they were far above marginal costs of building materials and labor inputs. Similarly, oil prices of $145 per barrel were explained as the result of supply and demand, even as demand eased and the cost of extracting oil from alternative sources remained below market prices. The consequences of ignoring common sense and basic finance in favor of these tenets of conventional wisdom contributed to the financial crisis. These two stories are examples of the same herd mentality that can lead analysts to mimic cost of capital components used by others rather than thinking carefully about the analysis.

Application of CAPM in Investment Decisions

The problems with CAPM and Adjusted Present Value (APV) methods of valuation that are drilled to MBA students in business schools are highlighted by attempting to apply the model with stock market data for real companies and alternative electricity generating plant investments. In addition to further illustrating problems with the CAPM, the analysis provides a modeling and information foundation for subsequent sections of the chapter where debt capacity finance and time series models are used as alternative approaches to measure risk and value. The example begins by evaluating data sources for estimating β, R_f, and $R_{m.}$ Next, adjustments to equity beta to de-leverage and re-leverage the beta are made so as to derive the WACC for application to free cash flow. Finally, ranges in cost of capital estimates that come out of the CAPM are applied in measuring the value of a natural gas combined cycle plant and a more capital intensive coal and hydro plant.

To come up with cost of capital for electricity plants, the first step is to try and find companies whose cash flow variability mimics the investment evaluated. A few companies that could provide useful stock price information involve firms that own generation plants and that are not subject to regulation. Companies in the industry that do not own any regulated generation or distribution include Dynegy, NRG, Mirant and Reliant. There are problems with using these companies because limited historic stock price data exists and many of the plants have fixed price contracts. Different approaches to measure beta for the selected energy companies are summarized in the table below. Beta statistics were obtained from Value Line, Finance.Yahoo!, and also computed independently using alternative statistical techniques discussed above. The table demonstrates a wide range of betas could be used depending on the sources used and the betas are unstable across periods. Highest betas are derived using the percent change in prices during the financial crisis and lowest are obtained from Yahoo and Value Line. Betas are computed from the stock price data are associated with equity values and it is typical to compute asset betas that adjust the comparative data for leverage as discussed in more detail below and in Appendix 3.

	Market Debt to Capital	Market Equity to Capital	Equity Beta: Yahoo Beta	Value Line Beta	Financial Crisis Beta	Monthly Beta	Weekly Beta
Dynegy	60%	40%	1.40	1.55	2.73	1.67	1.42
AES	78%	22%	1.52	1.20	1.59	1.50	1.25
RRI	22%	78%	2.14	1.70	2.86	2.24	2.38
NRG	47%	53%	0.95	1.20	1.24	1.02	2.81
MIR	12%	88%	1.23	1.05	1.59	1.28	1.02

	Market Debt to Capital	Market Equity to Capital	Asset Beta: Yahoo Beta	Value Line Beta	Financial Crisis Beta	Monthly Beta	Weekly Beta
Dynegy	60%	40%	0.56	0.62	1.10	0.67	0.57
AES	78%	22%	0.33	0.26	0.34	0.32	0.27
RRI	22%	78%	1.66	1.32	2.23	1.74	1.85
NRG	47%	53%	0.51	0.64	0.66	0.54	1.50
MIR	12%	88%	1.08	0.92	1.40	1.13	0.90
Average			0.83	0.75	1.14	0.88	1.02
Median			0.56	0.64	1.10	0.67	0.90
Equity to Capital			200%	200%	200%	200%	200%
Equity Beta			**1.12**	**1.28**	**2.19**	**1.34**	**1.80**

History of the merchant plant industry demonstrates how beta has not been a good measure of risk. After the industry was born in the 1990's, valuations declined dramatically after over-supply of natural gas plants occurred around the world in markets that had allowed merchant plants to enter the market. The case studies in chapter 1 of AES Drax, merchant peaking plants and Enron were all part of the meltdown that ultimately resulted in a decline in equity and debt value of $100 billion. Despite spectacular stock price swings of the merchant energy companies, the beta statistics were relatively low as shown in the table below. One of the most dramatic examples is Enron whose beta estimate from Value Line was only .95 and the beta estimate made by Bloomberg was .73. It is doubtful that when finance professors explain to all of the Enron employees that lost their net worth that their risk was diversifiable, one doubts that it would go over very well.

Equity Betas of Merchant Companies before 2000	Value Line	Bloomberg
AES	1.15	0.82
Enron	0.95	0.73
Dynegy	1.17	
Calpine	0.90	1.07

Even though there is a wide variation and many unresolved questions in estimating beta, the biggest problem with the CAPM is in coming up with the expected market risk premium. The table below summarizes actual market premiums from various valuation available sources at the time. Not surprisingly, this survey shows a wide range of opinions with respect to estimated risk premium, as the number is virtually impossible to estimate. The top number in the table was a number computed from using the Gordon model with dividend yields in the S&P 500 and presented by an expert in a court case. The 4.5% estimate was made by in the well known valuation book written by McKinsey consultants and used by many practitioners. The investment bank range was from a fairness opinion presented as part of a merger prospectus. After the financial crisis, investment banks have apparently increased the EMRP as demonstrated in the quote from the Pfizer/Wyeth merger:

> "The cost of equity was calculated using the capital asset pricing model, which is a theoretical financial model that estimates the cost of equity capital based on a company's "beta" which is a measure of a company's volatility relative to the overall market, a 6% market risk premium and a relevant predicted beta and risk-free rate."[23]

Using the table below, if the beta is 1.0 then the range from the highest R_m to the lowest R_m produces a cost of capital that varies by 6.5%. Therefore, even if the beta can be precisely defined, the problems in defining R_m leave the CAPM model with imprecise cost of capital estimates.

Once betas from stock market data have been computed, a series of adjustments are necessary to convert the data to and all-equity beta and then re-lever the beta according to market value of the debt and equity. These adjustments which are intended to remove the risk effects of debt leverage from beta are described in Appendix 3. As with measurement of beta, there are various different approaches to making adjustments for debt leverage. One of the points ignored by virtually all valuation texts that describe de-leveraging and then re-leveraging beta (which sounds a bit ridiculous) is that cost of equity does not change on a linear basis with debt leverage (this subject is discussed below in the context of Miller and Modigliani). Using the range in risk premium and all of the leverage, tax and β estimates, the table below shows the resulting range in cost of capital estimates. This shows that using different risk premium and beta assumptions described above results in a WACC range between 7.5% and 10.4%. In the next section I discuss how the range of cost of capital estimates affects the economic analysis of investment decisions.

Cost of Equity with Risk Free Rate of 4.5%

		Re-Levered Beta				
		Yahoo Beta	Value Line Beta	Monthly Beta	Weekly Beta	Financial Crisis Beta
EMRP		**1.12**	**1.28**	**1.34**	**1.80**	**2.19**
Academic Articles	**3.00%**	7.9%	8.3%	8.5%	9.9%	11.1%
Investment Bank Low Range	**4.00%**	9.0%	9.6%	9.9%	11.7%	13.3%
Investment Bank High Range	**5.50%**	10.7%	11.5%	11.9%	14.4%	16.6%
Citibank Recommendation	**6.00%**	11.2%	12.2%	12.5%	15.3%	17.7%
Litigation Case	**9.91%**	15.6%	17.2%	17.8%	22.4%	26.2%

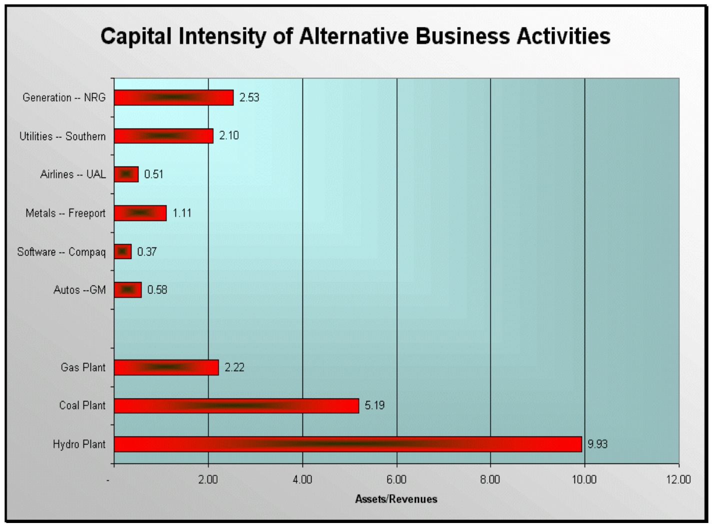
Capital Intensity, Cost of Capital and Value

The general ideal of capital intensity has been discussed on and off throughout the text. For capital intensive assets, the cost of capital component is a significant factor in the measuring the overall cost of the project relative to a less capital intensive investment where most of the costs are comprised of payroll, fuel and other items rather than in capital, the focus would be on evaluating payroll costs rather than evaluating capital costs. This section illustrates how the economic viability of assets with different levels of capital intensity is affected by the cost of capital assumption. To demonstrate how cost of capital measurement affects decision making, the value of a gas plant, a coal plant and a hydro plant is computed using alternative estimates. The analysis shows that the range in cost of capital has a dramatic affect on the investment decision – the low cost of capital range in the CAPM implies that capital intensive technology is more economic while the high cost of capital range suggests that the fuel intensive technology is more economic.

A statistic to measure capital intensity is the ratio of the capital investment as measured by assets to revenues. The higher the level of assets required to generate a dollar of revenues, the greater the capital intensity. The ratio of assets to revenues is driven by the capital cost of an asset relative to the operating costs, by the current assets relative to fixed assets and by lifetime of the capital investment. If operating expenses such as fuel are low relative to capital costs, or if the revenues are spread over a

[23] Pfizer S-4 SEC filing, 10 June 2009, page 78.

long time period, then the capital intensity is relatively high. Capital intensity of various firms and individual electric plants is illustrated on the graph below. The capital intensity of electric plants varies because different types of electric generating plants have a different relative mix of operating cost and capital cost. Projects with the highest capital-intensity are hydro plants and renewable energy technologies that have very high construction costs, low operating costs, and long lives. The least capital-intensive technologies are gas plants which have relatively high fuel costs and low capital costs.



Impacts of different cost of capital estimates on investment decisions that result from the CAPM are demonstrated using an exercise with three different generating technologies – a hydro plant, a coal plant and a gas plant. As shown in the accompanying graph, the hydro generating plant that is very capital intensive and the natural gas plant with significant fuel cost is less capital intensive. The cost structure of the hypothetical hydro plant from the standpoint of the tradeoff between capital costs and operating costs is similar to a renewable resource plant (wind or solar) or a nuclear plant.

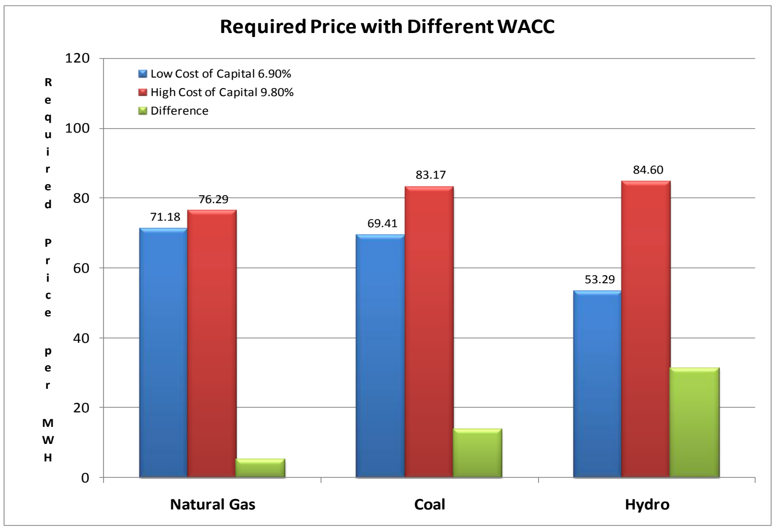
The case study compares the total cost of a capital intensive hydro plant with a less capital intensive gas plant and a coal plant. In this analysis total cost includes the opportunity cost of capital. The cost comparison depends on the assumed value drivers for each plant. Capital cost for the hydro plant is assumed to be $3,000/kW and operating and maintenance expenses are assumed to be $10/kW/year. The plant is assumed to have an economic life of 40 years. From an operating perspective, the plant is assumed to produce energy at a capacity factor of 50%. The natural gas combined cycle plant is used to represent less capital intensive investments. Capital cost of the gas plant is assumed to be $800/kW and the fixed operation and maintenance expense is assumed to be $25/kW/Year. The heat rate of the gas plant is assumed to be 7,000 Btu/kWh and the gas cost is $5.0 per MMBTU escalating at 3% per year. The variable operating and maintenance is assumed to be $3/MWH and the life of the plant is assumed to be 25 years. Finally, the coal plant is assumed to have a capital cost of $1,500/kW with a heat rate of 8,700, a fuel cost of $1.5/MMBTU and fixed O&M per kW of $30/kW.

Cost of Capital and the Investment Decision for Alternative Electric Generating Technologies

Two alternative methods can be used to integrate the cost of capital with the cost structure of the alternative electricity plants in order to make investment decisions. Either the required price can be derived using the cost of capital as an input, or the return can be computed top down from a given price projections. Think of the first approach as the bottom up method, and the second approach as the top down method. The bottom up method was described in chapter 1, where the first year market price necessary to achieve the internal rate of return -- equivalent to cost of capital -- was computed. The top

down method, the most economic alternative can be established by measuring the present value of free cash flows from forward price analysis. The bottom-up method is used to illustrate the CAPM impacts on the investment decision.

Using the bottom up method, the investment decision can be gauged by measuring which investment alternative yields the lowest required price given a cost of capital assumption. The low cost of capital estimated for competitive energy companies was 8.4%. This cost of capital implies the required price for the hydro plant is $35/MWH while the gas plant has a higher required price of $40/MWH. Therefore, at the low cost of capital the hydro plant should be selected as the best investment alternative.



The high range cost of capital derived from a higher assumed R_m resulted in a cost of capital estimate of 13.6%. Using this cost of capital, the gas plant appears to be a far better investment than the hydro plant – the gas plant has a required price of $49/MWH and the hydro plant has a cost of $67/MWH. The graph demonstrates that variation on CAPM cost of capital ranges completely changed the type investment decision. This means the cost of capital range also does not help us in deciding whether an investment should be made. The analysis demonstrates how important a precise measurement of cost of capital is to the investment decision. If the cost of capital is too high, no investments may occur. And if investments are made, projects such as capital intensive renewable technologies will be shied away from.

The discussion in this section demonstrated that application of the CAPM to measurement of risk associated with a generating plant has a variety of practical and theoretical problems. There is no ideal way to compute R_m, it is difficult to find project specific data for β, and adjustments to β for leveraged taxes are complex as shown in Appendix 2. Finally, and perhaps most importantly, the underlying theory of the model is questionable.

The case study demonstrated that differences in cost of capital can significantly affect technology choice. It also illustrated that the traditional cash flow technique does not offer much help in assessing the risk differences. Given all of the problems with CAPM you should keep in mind when considering alternative risk measurement approaches that even if alternative models are not perfect, they may still be preferable to the CAPM.

Converting Risk into Value Using Maximum Debt Capacity

The remainder of this chapter discusses alternatives to the valuation of debt and equity using techniques that derive the value of an investment from the debt and equity that are used to finance the investment. These techniques begin with the valuation of debt and extend the valuation to an entire investment. To see how debt and equity valuation can be the basis of valuation, consider the simple equation for the value of a business enterprise presented in chapter 2. The DCF method which uses beta and WACC along with operating assumptions measures enterprise value. Once the enterprise value is established, the equity value is computed from subtracting the value of debt used to finance the investment:

Equity Value = Enterprise Value – Value of Net Debt

The two approaches discussed below first establish the maximum amount of debt that can finance an investment and then add a remaining value of equity to establish the value of the investment. In a sense, the method re-arranges the above equation through beginning with valuation of securities on the liabilities and equity side of the balance sheet by assessing the risk of debt and equity cash flows rather than attempting to value assets through putting all of the cash flows together:

Value of Debt + Value of Equity = Value of Investment

The risk analysis process for raising debt to finance a project which involves credit analysis and measurement of debt capacity is used as the foundation to assess the risk and the overall value of an investment. This approach of beginning with debt value provides an alternative to the CAPM where value is derived from risk estimated from total free cash flow. The idea that valuation of debt raised to finance a project can provide insight to the overall value of the investment is founded on the notion that investors would like to maximize debt obtainable at a reasonable interest rate so that they can achieve a high rate of growth (i.e. rate of return) on the cash invested in the project. In the real world, it is often the case that if a project cannot raise much debt, the investment will probably not be made. Because of this, if one can determine the amount of debt that can be borrowed for an investment, one can back into the overall value of the assets that support the debt.

After demonstrating how an approach that computes debt and equity value from standard credit analysis techniques can establish value, the technique is extended in the next part of the chapter to compute debt capacity through valuing debt with time series models and Monte Carlo simulation. For both of these valuation methods that are founded on measuring the amount of debt that can be raised, the analysis cannot be boiled down to a single statistic such as beta. These methods instead involve a series of judgments with respect measuring the possibility that debt will default and the hurdle rate required by equity investors.

In addition to establishing the value of a project through measuring risk of the maximum debt that can be raised to finance a project, the analysis of debt capacity allows one to assess the effects of different type of contracts, hedges and insurance instruments that are put in place to limit risk on the value of an investment. Various tools used to manage risk including different types of contracts for constructing a plant; contracts for selling output at a fixed price; and, contracts for managing costs can be assessed by measuring how much debt a project can support. Consider for example, construction risks associated with a large project involving the possibility of cost over-runs.[24] As the cash flows associated with construction over-runs are generally not correlated to overall market fluctuations, the CAPM would come up with the nonsensical answer that risks of a construction cost over-run and/or a construction delay do not affect value. If you would tell investors who have been burned by cost-overruns in the A-380, the Olkiluoto nuclear plant in Finland, Eurotunnel or virtually any other large complex project that construction over-run and delay risk did not really exist because it could somehow be diversified, you would probably not receive a very polite response. Because these construction risks do not affect beta, the CAPM would not give you any guidance whatsoever in differentiating between two projects that have different types of construction contracts that protect debt and equity investors from cost over-runs, delay and technical problems assuring that an construction can be completed.

[24] There is much evidence that an optimism bias exists in estimating construction costs. See the U.K.

The manner in which debt capacity and can measure how risk affects value can be demonstrated by considering the way in which construction risk is evaluated. It is possible that an investment can employ a fixed price construction contract between investors in a project and construction company. Assume the contract also has liquidated damages that force the construction companies to provide monetary compensation for delays or projects that do not have the technical specifications promised. This type of contract known as a lump-sum, date certain turnkey contract "LSTK" allocates risk to the construction company has very different value implications than a cost plus contract which leaves risks of cost over-runs, delays and poor quality with debt and equity investors. The risk transfer between investors and the construction company from using an LSTK contract is of course generally not free to the investor. For example, in the case of construction of natural gas plants, a very rough rule of thumb is that a plant with an LSTK contract will have a 10-20% higher expected cost than a plant built with a cost plus contract. Although the expected cost of the contract is higher, the fact that the construction cost over-run and delay risk are reduced may mean that signing an LSTK contract increases value. Computing enterprise value using the CAPM would not be able to assess this value tradeoff between a cost plus and an LSTK contract and would imply that the cost plus contract should always be used as the construction risks are diversifiable and therefore the LSTK contract provides no value. On the other hand, if the presence of the LSTK contract increases debt capacity and thereby allows investors to realize a higher rate of return on their investment, the increase in debt capacity offsets the higher cost of the LSTK contract. Using the debt capacity analysis, if one can assess the difference in the amount of debt a bank is willing to lend because of the two different contracts, then we surely have a much better tool than the CAPM to translate risk into value.

As explained below, professors at most business schools most would probably not agree with the notion of using a debt instrument to evaluate the overall risk of a project. This technique violates the famous Miller and Modigliani thesis as well as the theory behind the CAPM, both of which resulted in Nobel prizes. The method also cannot be boiled down to a seemingly elegant statistic. Using credit analysis and debt capacity concepts to measure the overall value of an investment better reflects the complexities of how real investments are made. Rather than looking backward at historic data, and the impossible to measure market risk premium in making valuations, it uses a series of measures used by financial analysts who assess the value of projects.

The remainder of this section walks through how risk analysis of the debt issued in a project can be used to measure the overall value of an investment. The discussion begins by recounting some problems with finance theory and typical application of the ideas of Miller and Modigliani. Next, the manner in which debt capacity and credit analysis affects real world decision making is described. After describing the theory of credit analysis, an example of computing debt capacity to valuation of an investment with and without a contract is presented. Finally, issues of how equity value can be derived after the debt capacity is obtained and problems with the approach are addressed.

Debt Capacity and Finance Theory Developed by Miller Modigliani

The notion of using debt capacity to value an investment introduced above seems to directly conflict with the Miller and Modigliani thesis which has led to the idea of focusing valuation on free cash flow rather than equity cash flow. In the late 1950's Merton Miller and Franco Modigliani developed their famous propositions which demonstrated the simple idea that the value of an investment is determined by things that affect the way it operates and not by the manner in which it is financed.[25] Using this thesis, business schools have emphasized separation of the financing decision from overall investment valuation for many years. The fundamental point Miller and Modigliani made is that it is not how you slice the pie between debt and equity, but factors that affect the overall size of the pie that matter. Free cash flow (the sum of equity cash flow and debt cash flow as explained in chapter 2) defines value of an

[25] Miller, Merton and Modigliani Franco, "The Cost of Capital, Corporation Finance and the Theory of Investment", American Economic Review, 1958.

asset while debt and equity are simply claims on that free cash flow. With the exception of the tax deduction for interest and bankruptcy costs, financing the asset involves slicing the pie, not the size of the pie. Attempts to assess risk and cost of capital through evaluating the instruments that finance assets are counter to this basic theory.

The ideas developed by Miller and Modigliani were not only brilliant in their simplicity, but also by the way they introduced arbitrage as a tool in analysis of financial issues. Using arbitrage in valuation allows one to indirectly measure the value of something instead of trying to directly measure how risks are translated into value as illustrated in the discussion of the Arbitrage Pricing Model above. Through using arbitrage analysis, one can attempt to create an artificial risk-free portfolio and then indirectly measure value by realizing that risk free cash flows are discounted at the risk free rate. The arbitrage concept introduced by Miller and Modigliani involved demonstrating how investors can engage in one of two strategies:

(1) They can buy shares of company that has no leverage and then borrow money on their own behalf to buy the shares; or,

(2) They can buy shares of a leveraged firm.

Since these two strategies yield the same cash flow, the assets that produce these cash flows must have the same value, proving that the value of the firm is not affected by debt leverage. The idea of using arbitrage to create equivalent cash flow using two strategies has changed the way we think about many financial issues raging from risk neutral valuation discussed above to option price analysis addressed in the next chapter.

One of the main implications of Modigliani and Miller is that the required equity return increases with debt leverage. On the other hand, without taxes, the overall cost of capital is independent of the capital structure and is only affected by risks associated with operating cash flow because the total value of the firm does not change with leverage. This implies that in making investment decisions, investors should not use an equity return criteria but they should instead compare the overall WACC which is independent of the capital structure to the return on the overall free cash flow. The Miller and Modigliani ideas were applied in the 1980's and 1990's by consulting firms when they developed a concept known as economic value added ("EVA"). Advocates of this idea asserted that financial performance of corporations, divisions of firms and managers should be tied to free cash flow relative to overall cost of capital that ignored the debt leverage and did not concentrate on things like earnings per share. Financial theorists and EVA advocates probably would cringe at the notion that investment decisions should be driven by analysis of debt capacity -- the proposition of this section. For example, in the book *EVA*, Al Ehrbar states that the equity return changes with leverage so that the overall return remains about the same:

> *"What's more, the required rate of return to equity investors rises with the degree of leverage in a company's balance sheet. Two companies in the same industry will have very different required rates of rate of return to shareholders if one has no debt and the other has borrowed to the hilt."*[26]

Most finance textbooks these days discuss the Modigliani Miller idea that the cost of equity capital is simply the result of leverage and the inherent risks associated with operating assets. Without taxes or bankruptcy costs, the texts assume the overall cost of capital and the debt costs stay constant, while the cost of equity increases with leverage. Given that the overall return (and WACC) is driven by volatility and beta associated with free cash flow, the equity return can simply be backed out from a traditional weighted average cost of capital table. This is illustrated in the two tables below which assume no taxes and debt ratios of 50% and 75% respectively.

[26] See Ehrbar, Al, *EVA: The Real Key to Creating Wealth*, John Wiley and Sons, page 42.

WACC Table				WACC Table			
	Percent of Captial	Cost of Captial	Weighted Average Cost		Percent of Captial	Cost of Captial	Weighted Average Cost
Debt Cost	75%	5.0%	3.8%	Debt Cost	50%	5.0%	2.5%
Equity Cost	25%	25%	6.3%	Equity Cost	50%	15%	7.5%
Overall Weighted Cost of Capital			10%	Overall Weighted Cost of Capital			10%

In the table above, the 10% overall return or WACC is determined by inherent risks in the operating assets, while the return on equity is backed out from the other figures (it is computed as the overall return less the debt WACC divided by the equity percent). On the left panel, with 75% debt in the capital structure, the cost of equity that falls out of the equation is 25%; while the right-hand side with 50% debt in the capital structure produces a required return of only to 15%. Differences in the cost of equity also correspond to the increased risk incurred by equity holders from the effects of leverage. The equation to back out the equity cost of capital is simply:

Equity Cost of Capital = (Asset Cost of Capital – Debt Percent x Debt Cost)/Equity Percent

A corollary to this is the notion that the beta increases with leverage which has already been introduced. The idea comes from the fact that beta that is computed from equity prices and represents beta associated with the observed cost of equity capital. Given the asset beta, if there are no taxes, the beta associated with overall cash flow is:

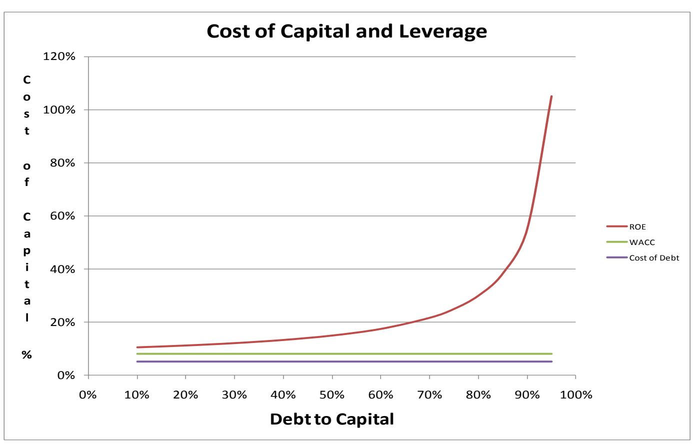
$$\text{Beta}_{\text{assets}} = \text{Beta}_{\text{Equity}} \text{ x Equity/Capital} + \text{Beta}_{\text{Debt}} \text{ x Debt/Capital}$$

If the debt beta is zero, which is generally assumed, then equity beta can easily be derived from the assets beta as illustrated below:

$$\text{Beta}_{\text{assets}} = \text{Beta}_{\text{Equity}} \text{ x Equity/Capital or, } \text{Beta}_{\text{Equity}} = \text{Beta}_{\text{Assets}} \text{ x Capital/Equity}$$

Investment bankers often first compute the equity beta for a sample of companies, and then compute the average asset beta (using the market value of equity and the market value of equity plus debt.) Next the asset beta is "re-levered" to compute the equity beta.

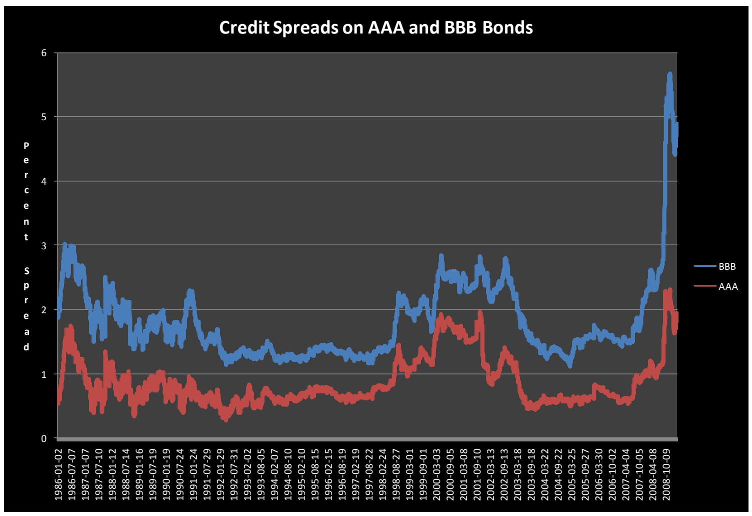
The idea that beta increases in a non-linear fashion with leverage and that cost of equity is derived from overall return and leverage is often shown on a graph that displays leverage on the x-axis and cost of debt, equity and WACC on the y-axis. This type of graph is shown below where the implied cost of equity reaches more than 100% at high levels of debt leverage:



The fact that this graph continues to be presented in finance texts and that the equity beta/asset beta adjustment continues to be used by investment bankers is almost scandalous. It is just as wrong to assume that the equity beta increases in proportion to the equity to capital ratio as the equity beta has the same pattern as the cost of equity. The big problem is that in the real world, the cost of debt increases with leverage because the credit spread is directly related to firm's debt leverage. The cost of debt capital can be computed from the formula:

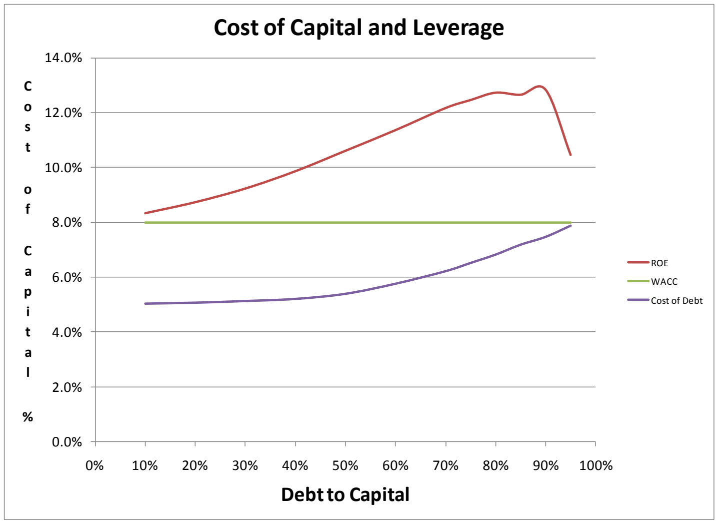
Cost of Debt = Risk Free Rate + Credit Spread

Credit ratings, probabilities of default, and credit spreads of course do not remain constant as the level of debt leverage changes. Higher leverage increases risk and results in a lower credit rating, all else being equal. The manner in which credit spreads change with the bond rating is shown in the graph below. Since 1986, the first year shown on the graph, credit spreads on AAA bonds ranged from less than a half a percent to almost two percent during the financial crisis. Credit spreads for investment grade BBB bonds varied between three percent and declined to almost one percent just before the financial crisis.[27]



An analysis that incorporates credit spreads into the WACC analysis with changing leverage is presented in Appendix 4. The appendix evaluates how the credit spread on debt changes as the ratio of debt to capital changes assuming different levels of volatility in operating cash flow using Monte Carlo simulation. Results of the credit spread analysis that change the level of leverage are shown in the graph below, where the overall WACC remains constant, but the cost of debt increases because of the higher probability of default that arises with more leverage. This graph demonstrates the dramatic effect of recognizing increasing credit spreads in the typical leverage versus cost of capital graph. Once the logical and quite obvious change in debt cost is included in the WACC table, the resulting cost of equity has a completely different shape. Now, rather than the cost of equity capital increasing in proportion to the leverage, the cost of equity stabilizes and then even falls as the option characteristics of equity dominate the effect of increasing variability of cash flow. The implication of this analysis that includes credit spreads is that one cannot assume the return on equity continues to increase using the simple formula from the WACC calculation as leverage grows. In addition, the common technique used by bankers to de-leverage and re-leverage betas on a straight line basis cannot be applied when credit spreads are accounted for in the analysis. The equity beta can decrease with increases in leverage just as the cost of capital decreases as shown in the graph below.

[27] The data is downloaded from the St. Louis Fed website. A file that allows you to download from the website is included on the CD.



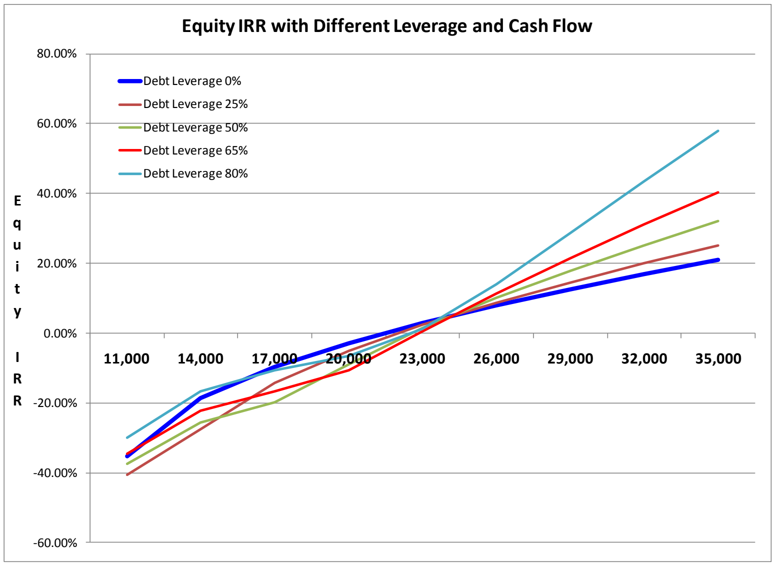
The Modigliani and Miller thesis should have created a dilemma for academics who peek into the real world. If financing of an asset is irrelevant for valuation, why then, from the standpoint money raised for capital investment, does the amount of debt that can be raised for an investment seem so important. If debt did not affect the type or amount of physical investment, then the implications to corporations from changes in bond ratings should not have dramatic effects on corporate strategy or share prices. If the Modigliani and Miller propositions (without taxes) that the value of a firm is independent of its capital structure in equilibrium in perfect capital markets, nobody should be very concerned about the dramatic errors made by credit rating agencies nor the high level of credit card debt accumulated by American consumers.

Debt Capacity, Equity Returns and Real World Investment Decisions

Use of debt capacity to measure cost of capital can be illustrated by considering the hypothetical decision making process used by the financial analysis department of a company that makes capital intensive investments. Start with the notion that management has a rate of return criteria, where only projects that have an IRR of above 15% are approved for investment and projects that have an IRR below 15% are not (this directly conflicts with the Miller Modigliani, but we will see it is not irrational). For most companies this rate of return is measured using equity cash flow rather than free cash flow, due to corporate objectives related to earnings per share (“EPS”) growth. As long as the growth rate in free cash flow from the project -- the project IRR -- is expected to yield a higher rate of return than the after tax cost of debt, the equity return can because increased if more debt is used to finance the asset. (Magnifying asset returns to increase equity return is the where the term leverage comes from). Say that because of the reluctance of bankers to take credit risk, debt cannot be raised for the project. In this case the equity return criteria will probably not be met and it is unlikely that equity investors will agree to make the investment. On the other hand, if a lot of debt can be raised because bankers accept the project risk, the equity IRR will exceed 15% if the expected cash flow is achieved and the investment will be made. In this case the amount of debt does directly affect the investment decision. Indeed, the investment is driven by the amount of debt that can be raised rather than by the beta of the project or the net present value of cash flows discounted at the WACC.

The relation between the equity IRR, the project IRR and debt leverage is shown in the graph below. The horizontal axis shows variation in cash flow and the five lines present variation in the Equity IRR’s using different debt financing assumptions. For the case with no debt financing, the internal rate of return on equity is the same as the project IRR – free cash flow is the same as equity cash flow when there is no debt. When debt is used to finance the project, the equity cash flow variability increases relative to free cash flow variation; particularly if the realized cash flow is greater than the expected cash flow. The higher variance in equity cash flow with more debt leverage is the traditional reason that the

cost of equity capital is assumed to be higher than discount rate applied to the un-leveraged project cash flow. Note that with low levels of cash flow the variation in equity return due to leverage is less. This is due to the option characteristics of debt and equity; in cases with high leverage, the low level of investment limits the equity IRR exposure.



The above graph illustrates one of the reasons that high debt is important for equity investors – they achieve more upside while lenders take some of the downside. More importantly, the idea of presenting different levels of leverage independent of cash flow variability in the graph distorts the most essential point that there is a connection between the level of debt financing and the variability in cash flow. If the debt financing depends on the variability of cash flow, then the amount of leverage itself defines risk. If an investment is made because at the expected level of cash flow results in an equity IRR that exceeds the management criteria, then risk has been defined by the leverage and not the other way around.

Focusing on equity return means the investment decision can be driven by debt capacity rather than by project IRR. For example, assume one project has a project IRR of 8.5% while a second has a project has a lower project IRR of 7.5%. Say the expected equity return for the first 8.5% project that can support 50% leverage is 9% while the projected equity return for a second project 7.5% with 80% leverage and longer term debt financing is 15%. Instead of spending a lot of time working about WACC, net present value of free cash flows and the project IRR, in the real world, the second project with higher equity IRR and lower project IRR will generally be chosen even if the beta on both projects is somehow determined to be the same. Valuation of the project is driven by equity cash flow rather than free cash flow. In this example, the real world manager is not making investment decisions on inappropriate criteria. Rather, this process means that the overall cost of capital is determined somebody outside the company -- bankers and financial professionals -- who are not developers or managers in the firm making the investment, or equity holders with a bias toward making the investment. These bankers who influence equity IRR do not have a vested interest in making the project look good.

Maximizing Debt Leverage Subject to Investment Grade Constraint

The idea of using debt capacity to measure risk makes the assumption that equity investors want as much debt as possible in order to obtain maximize the growth rate on their cash investment. To demonstrate why the incentive to maximize leverage is an important issue in raising debt, return to the two project example introduced above. Let's say that the level of debt did not matter to owners of the

project -- for example the same investment would be made whether the project had 40%, 60% or 80% leverage because of evaluating the project IRR relative to the WACC. In this case, debt capacity would not tell us anything about risk and value because there is no objective function which suggests that as much debt as possible should be used and there is no risk constraint from the banks that limits the debt issued. Without the incentive to maximize debt, a project with very volatile cash flow may have leverage of 80% while a project with safe cash flows may use leverage of 40%.

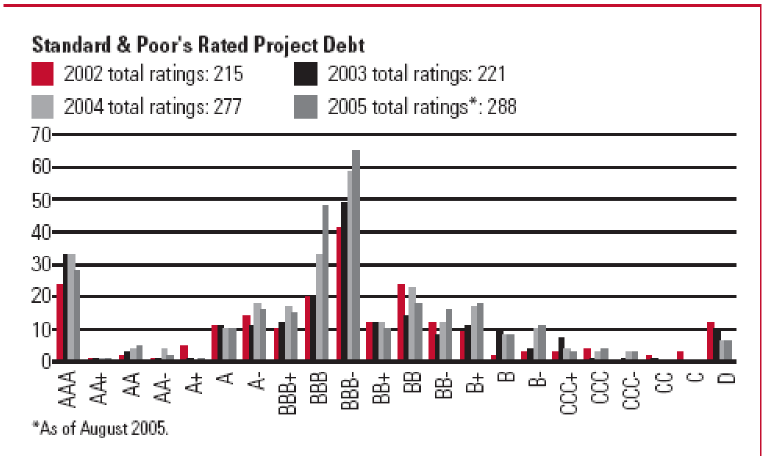
As explained above, the first Miller Modigliani proposition demonstrated that, in a world without taxes, debt capacity should be irrelevant to an investment decision. If leverage was not pertinent to the value of an asset and the decision to make an investment, the study of debt capacity would not be helpful in risk assessment, cost of capital measurement and valuation of an asset. However, Miller Modigliani themselves and others have recognized that there are a variety of reasons for debt is desirable. Appendix 5 discusses some theory associated with each of these incentives to maximize debt in the context of project finance transactions. The reasons include lowering income taxes, sending positive signals to investors, increasing earnings per share, and the taking advantage of low direct costs of bankruptcy. The tax savings incentive which is particularly important for capital intensive assets is summarized by the following statement made by Merton Miller in 1988:

> *"Many finance specialists remain unconvinced that the high-leverage route to corporate tax savings is either technically unfeasible or prohibitively expensive."*[28]

While taxes are often cited in the literature as the principle incentive to use debt financing, the most important incentive discussed below is not taxes, transaction costs or other theory, but the information provided on risk by lenders who put up their money and who should have a strong incentive to develop objective risk analysis. Further, the risk analysis performed by lenders concentrates on downside scenarios, which is the important side of risk to all investors – i.e. what will happen to the investment when the world falls apart.

In practice, the incentive to use more debt financing in financing a transaction is not unlimited, but is constrained by an unacceptable credit rating. The incentive of management can be re-phrased as a desire to push up the amount of debt financing until a credit rating of just above or just below investment grade in obtained (a rating of below BBB- is defined to be junk using Standard and Poor's terms). This is demonstrated by the graph below which presents the number of project financings arranged by bond rating. The graph shows that the greatest number of project finance transactions had a rating of BBB-, the lowest bond rating that is not investment grade. The idea of ranking debt does not only apply to bonds, but also to bank debt. Bankers have similar rating systems that may use numbers or letters and one of the numbers or letters roughly corresponds to the minimum investment grade ranking. If one would rank the number of project financing transactions at banks, the result would be very similar to the Standard and Poor's graph where the largest number would be concentrated in a range just below investment grade.

[28] Miller, Merton, "The Modigliani-Miller Propositions After Thirty Years", Journal of Economic Perspectives, Fall, 1988.



The reason that the objective function is to issue as much debt as possible with a near investment grade rating classification involves the cost and difficulty in raising financing if ratings fall to junk levels. If ratings are below investment grade it may be difficult to secure financing and the interest rate on the debt generally increases by a large amount. When observing the credit spreads and the bond ratings, the largest differential is generally between the BBB and the BB rating. The credit spread differential and the difficulty in securing financing was demonstrated after the financial crisis. During the boom before the crisis, the credit spread on BB debt was very low by historic standards – below 3%. After the crisis the spread became very high if it was even possible to secure junk debt with credit spreads stabilizing at to 6% while the spread on investment grade bonds was about 3%. Therefore, when the idea of maximizing debt capacity is discussed, what is really meant is that the amount of debt is maximized subject to the constraint that it remains close to investment grade.

If two debt issues have the same level of safety as measured by the debt rating (or some other criteria such as internal bank ratings) then the rating process can be reverse engineered so that differences in debt leverage are be used to measure the operating risk of projects. When comparing the two projects, the project with riskier free cash flow will be able to support less comparably rated debt. For instance, assume the two projects have an rating of "BBB-/Baa2" which is just above investment grade, and that the first project has higher debt leverage than the other project. Since the first project was able to obtain the same rating and also higher debt leverage, one can infer that the risks associated with its inherent free cash flow are lower than the second project which was not able to achieve as high a rating.

Measuring Debt Capacity with Credit Analysis

Given the incentive of maximizing leverage subject to an investment grade constraint, the issue of how to determine a rating classification must be addressed in order to apply the debt capacity approach to measuring risk. The process of rating debt, which is a proxy for the probability of default, is determined through credit analysis, meaning that to understand valuation of an investment one must work through the credit analysis process. Many of the valuation mistakes discussed in chapter 1 were mistakes in credit analysis including the sub-prime crisis, Eurotunnel, AES Drax and Enron Dabhol. In particular, the dramatically poor credit analysis of mortgage loans that precipitated the financial crisis has resulted in revisiting the appropriate analysis that should be conducted for credit analysis. Mistakes included over-reliance on computer models both for individual housing loans and for complex structured debt products that aggregated the loans; very poor due diligence of individual loans performed by people who had no experience and no incentive to question transactions; distorted incentives which resulted in focus on fees rather than analysis of the probability of default; fraudulent appraisal of homes which provided collateral for the loans; and use of loan to value ratios rather than cash flow to debt service ratios in sizing debt and ignorance of economic fundamentals. An active debate began with respect to how much mathematical modeling versus business judgment should be used in the process of

classifying loans and estimating the probability of default. The approach to debt capacity in this section uses more judgment while the discussion in the next section addresses debt capacity from use of mathematical analysis in the credit analysis process.

Traditionally, credit analysis has involved developing standards to classify loans such as bond ratings from credit rating agencies and credit scoring in banks. If credit analysis is working, then the bond ratings and credit scores should be correlated to the probability of loans not being repaid (bond rating agencies such as Standard and Poor's, Moody's and Fitch regularly publish correlations between their bond ratings and the probability of default to demonstrate the validity of their risk rating systems.) Credit scoring systems generally involve assessing financial ratios such as the debt service coverage ratio and determining whether key credit risks can be mitigated. Credit risk also accounts for the collateral and the chances of recovering money once things go wrong.

Old fashioned credit analysis involved the so-called five C's of credit that included (1) character, (2) capacity, (3) capital, (4) collateral and (5) condition. In contrast, mathematical analysis uses time series models, option price analysis and Monte Carlo simulation to directly compute the probability of default and the loss given default. Many of the mathematical and traditional concepts are just different ways of analyzing the same thing. For example, the capacity of a loan to repay is analogous to the volatility in option pricing models; the capital or the amount of leverage is the same as the strike price used in mathematical analysis; and the collateral is comparable to measurement of the loss given default computed with statistical analysis. On the other hand, the character of management has been an important issue in many bankruptcies including Enron and the sub-prime crisis and cannot be evaluated using pure mathematical analysis.

Application of traditional credit analysis in computing the size of debt involves using financial ratios to determine how much buffer is acceptable for a given loan. Ratios frequently used are the debt to capital ratio; the ratio of debt to EBITDA; and the debt service coverage ratio. The debt to capital ratio or loan to value ratio is often of limited use because it depends on the valuation of assets rather than the cash flow that is available to repay the debt. For example, after a merger, the value of assets often changes by a wide margin because of purchase accounting where the asset valuation is driven by the how much is paid for acquiring a company. If the debt level increases to finance a merger, but the purchase price relative to the exiting book value increases even more, the debt to capital ratio decreases. The ability of cash flows to service debt can better be assessed by the ratio of debt to EBITDA which measures how many of years of EBITDA or operating cash flow it would take to repay the total debt (with no capital expenditures, taxes or interest expense). The higher the ratio of debt to EBITDA, the longer time it takes to repay debt. Finally, the ratio of cash flow to debt service measures how much buffer there is in cash flow. If the ratio is 1.0, then there is no buffer and cash flow is just covering the amount of money that must be paid to service the loan (interest expense plus required repayments are defined as debt service). If the debt repayments are smooth over time (with no bullet maturities) then testing how low cash flow can go before the debt service coverage falls to 1.0 can be an effective way to evaluate credit risks. The single most important element in credit analysis of project finance transactions is assuring that debt service can be paid, meaning the debt service coverage ratio is above 1.0, in a downside scenario.

The problem with applying historic or prospective financial ratios in credit analysis and classification of loans is that the acceptable level of a ratio depends on the operating risk of a company. For example, if a company is operating in a very competitive and capital intensive industry with volatile cash flows, a debt to capital ratio of 50% may imply the an investment grade rating cannot be achieved. On the other hand, if the company has a strong position in a very stable industry, a debt to capital ratio of 70% may be acceptable for achieving investment grade. The manner in which ratios can be used to classify credit ratios is illustrated in the figure below from Standard and Poor's. The business profile shown on the left side of the ratio is a measure created by S&P of the business risk. A ranking of 1 implies very safe and stable operating cash flow while a ratio of 9 implies the opposite. If one knew the business profile of a company and the series of financial ratios, then one could go down and across in the tables to find the implied bond rating. For example, if the business profile is four and the cash flow divided by debt is 15% then rating should be BBB. All of this would all be quite straightforward if S&P was

transparent in the way it determines business profile. In fact, the opposite is true and it is generally not possible to determine how business profiles are computed and it is also expensive to acquire data from S&P without paying fees. Notwithstanding the S&P lack of transparency, the table does illustrate the general manner in which financial ratios can be combined with judgment with respect to operating risk to determine credit classification.

S&P Benchmark Ratios

Funds from Operations Interest Coverage (x)

Business Profile	AA		A		BBB		BB	
1	3	2.5	2.5	1.5	1.5	1		
2	4	3	3	2	2	1		
3	4.5	3.5	3.5	2.5	2.5	1.5	1.5	1
4	5	4.2	4.2	3.5	3.5	2.5	2.5	1.5
5	5.5	4.5	4.5	3.8	3.8	2.8	2.8	1.8
6	6	5.2	5.2	4.2	4.2	3	3	2
7	8	6.5	6.5	4.5	4.5	3.2	3.2	2.2
8	10	7.5	7.5	5.5	5.5	3.5	3.5	2.5
9			10	7	7	4	4	2.8

Funds from Operations to Total Debt (%)

Business Profile	AA		A		BBB		BB	
1	20	15	15	10	10	5		
2	25	20	20	12	12	8		
3	30	25	25	15	15	10	10	5
4	35	28	28	20	20	12	12	8
5	40	30	30	22	22	15	15	10
6	45	35	35	28	28	18	18	12
7	55	45	45	30	30	20	20	15
8	70	55	55	40	40	25	25	15
9			65	45	45	30	30	20
10			70	55	55	40	40	25

Total Debt to Total Capital (%)

Business Profile	AA		A		BBB		BB	
1	48	55	55	60	60	70		
2	45	52	52	58	58	68		
3	42	50	50	55	55	65	65	70
4	38	45	45	52	52	62	62	68
5	35	42	42	50	50	60	60	65
6	32	40	40	48	48	58	58	62
7	30	38	38	45	45	55	55	60
8	25	35	35	42	42	52	52	58
9			32	40	40	50	50	55
10			25	35	35	48	48	52

The above table is generally applicable to on-going corporations where the debt maturities are not calibrated to the prospective operating cash flow. For project financed transactions where debt is issued to finance single assets, the debt service coverage (DSCR) is generally the primary focus of credit analysis. In discussing debt service criteria and other general benchmarks for project finance, one can often obtain data from discussions with bankers for prospective minimum DSCR's assuming expected levels of cash flow are obtained. Sometimes one can also obtain the lower benchmarks for a downside case. For example, for project financed wind projects with fixed tariffs, the typical DSCR is 1.4 – 1.5 times for a base case where using the expected wind speeds or a "P50" case. In the downside case with lower wind speeds where there is a 90% probability that the wind speed will be higher than the forecast level, a 1.15 – 1.2 times DSCR could be used.

While generally not directly stated, these benchmarks are generally applicable to loans that have a marginal investment grade rating or just below an investment grade rating. In the case of project financings, the required buffer in expected cash flow, or the amount by which the debt service coverage ratio should exceed 1.0 depends on the industry, contract types, cost structure, physical reserves in the case of oil and mining, covenants in loans and other factors. The table below illustrates ranges in debt service coverage ratio standards for various different types project finance transactions and illustrates that there is more transparency in project finance DSCR criteria than in corporate finance ratings for which one must somehow guess the business profile classifications.

Average Debt Service Coverage Ratios for Different Project Types (Times)					
	Target	**Aggressive**	**Risky Project**		**Aggressive versus Risky**
Infrastructure	1.3	1.2	1.8		Cash Traps, Availability Payments, Existing Traffic Established
Power	1.3	1.2	1.6		Length of PPA, Merchant Exposure, Off-taker Quality
LNG	1.9	1.4	2.5		Offtaker quality; contracts
Oil and Gas	1.7	1.5	2.0		Reserves after loan, PLCR
Mining	1.6	1.4	2.0		Reserves after loan, cost structure
Telecom	2.0	1.7	2.8		Corporate credit ratios

For project finance transactions, one can often derive precise information about risk through evaluating DSCR data for alternative existing transactions. To illustrate this process, a number of electricity generating projects with and without long-term revenue contracts that had publicly rated debt before the credit problems in 2001-2002 are shown in the table below. The table presents debt leverage, debt service coverage and debt terms for publicly rated electric generating projects that had debt rated by either Moody's or Standard and Poor's. In addition to presenting debt leverage, the term of the debt is also presented in the table because debt capacity is not simply measured by the amount of debt issued relative to the total amount spend on a plant, but also by how long the debt is outstanding. The table demonstrates that projects without long-term contracts have far higher risk. Use of DSCR's and other debt terms will be used to measure investment risk and overall required return on capital for alternative projects.

Information from actual projects in the table below shows that projects with long-term contracts (less revenue volatility) have higher leverage, lower debt service coverage and longer terms than projects that are exposed to merchant pricing which results in more volatile cash flows. Applying data from the table in measuring debt capacity allows one to compute the effects of different contract structures on the value of a plant and the acceptable pricing of a contract.

Project	Rating	Contract	Capacity MW	Debt	DSCR	Capital	Debt/Capital	Debt/Capacity	Debt Term
Projects with Contract Pricing									
Attala Generating Co. LLC	BBB-	25 Yr Tolling	526	359	1.52			682.03	37.5
AES Ironwood	Baa3	20 Year	705	324	1.48	359	90%	459.01	26
AES Red Oak	Baa3	20 Year	832	384	1.52	440	87%	461.54	30
East Coast Power	Baa3	Long-Term	953	1,156	1.30	1,333	87%	1,213.01	
Indiantown	Baa3	30 Year PURPA	330	626	1.48	767	82%	1,896.97	
Midland Cogeneration Venture	Ba2	25 Year PURPA	1,240	1,929	1.24	2,128	91%	1,555.65	
Sithe Independence	Baa3	40 Year PPA	1,000	698	1.40	761	92%	698.00	25
Sutton Bridge	Baa3	15 Year Tolling	790	317	1.38	356	89%	401.27	
Tenaska Georgia	Baa3	29 Year	936	275	1.27	310	89%	293.80	30
Average Contract					**1.40**		**88%**		**30**
Projects with Merchant Pricing									
AES Eastern Energy L.P.	Ba1	Merchant	1,268	650	2.26	1,091	60%	512.62	29
PPL Montana	Baa3	Merchant	1,260	338	2.50	629	54%	268.25	15
Mission Homer City	Baa3	Merchant	1,884	1,092	2.20	2,044	53%	579.62	23
Orion Holdings	Ba3	Merchant	5,926	2,431		3,653	67%	410.23	10
Average Merchant					**2.32**		**58%**		**19**

Equity Valuation and Required Return on Equity

Establishing the amount of debt that can be raised to finance an investment does not mean that the equity value can be ignored in establishing the total asset value or the overall required rate of return. With a lot of debt issued to finance the investment of a project, the relative importance of equity is indeed reduced, but the cost of equity still must be measured, which seems to mean that all of the problems with the CAPM are still around. The CAPM problems are further complicated because of the implicit assumption in the CAPM, discounted cash flow and most standard financial analyses is that cash flows follow a normal distribution. However when debt is used to finance an asset or a company, it is well known that equity has characteristics of a call option. Because the most that can be lost is the original investment, but large upside returns can be realized, the returns to equity investors have a skewed distribution if debt is part of the capital structure. This has already been demonstrated in the previous chapter and will be further elaborated on below.

Because of the option characteristics of equity, it could easily be the case that the cost of equity is lower than the cost of debt. To see this consider a simple scenario where an investment of 1,000 is made and the future cash flows are either 1,800 in an upside case, or 500 in a downside case for one period in the future. Further, assume the risk free rate is zero. If 900 is lent to the project, and the interest rate is 50%, than the lender either receives 900 X 1.5 or 1,350 in the upside case or 500 in the downside case for an expected value of 900. This is the same amount as the lender would receive if he invested in a risk free security that paid 1,000 in the next period (recall that the risk free rate is zero). In the simple case, the cost of debt is 50%, but if this were used to value the entire project, the project's value would only be .5 x (1,500/1.5) + .5 x (500/1.5) = 600. In a risk neutral world however, the project has a value of 1,000 by definition which is verified by the fact that the lender is fully compensated with a credit spread of 50%. To back into the overall cost of capital, the expected value of cash flows can be divided by one plus the cost of capital implying an overall cost of capital of 15%. This means the debt cost of capital is not appropriate for use in valuation. It also demonstrates that the cost of debt can be greater than the cost of equity and the unleveraged cost of capital.

Rather than preparing a whole lot of analysis on the cost of equity capital in project finance investments, one can survey equity sponsors to find out what are their minimum required rates of return. This is analogous to the manner in which investment banks and pension managers are surveyed in order to establish the EMRP discussed above. If equity investors in different projects have similar return requirements, the survey approach is then no worse than the CAPM, which boils down to the same thing because of the impossibility in measuring the EMRP. With a lot of debt in the capital structure, at least the relative importance of the equity return is minimized.

When surveying equity sponsors in projects, one generally finds similar minimum required returns for projects in different industries with different contract structures and debt levels. The idea that projects with different levels of debt would have similar equity return requirements runs directly counter to the Miller Modigliani ideas discussed above, but it makes sense. Lenders provide an equilibrating mechanism that leaves equity cash flow with reasonably similar risk after debt capacity has been established from risk in operating cash flow. Once the debt capacity is determined, the variability in remaining equity cash flows that provide a margin of comfort to the lenders should be similar for projects with different levels of leverage. This margin of comfort (or buffer) defines the risk faced by the equity holders themselves. For example if one project has a high level of operating risk such as merchant plants in the table above and cannot support much debt while a second project such as the contract plants in the table above has lower operating risk but is able to support more leverage, the residual equity risk should be similar. Therefore, it is not an accident, nor a misapplication of financial theory that leads equity investors to have similar return on equity requirements for projects with different debt leverage.

Instead of surveying investors, one can also back into required returns through creating models of projects that are bid on a competitive basis. When companies compete to bid on a toll road, an airport, an electricity plant or another project with a contract, the tariff level along with the debt capacity defines the equity IRR for equity sponsors. The equity IRR for the project that wins the bid with the lowest tariff should then define the minimum required return. The chart below shows the equity IRR for various toll

way projects from 1994 to 1996. Since 1999, most of the returns were between 12% and 13% demonstrating the stability in returns across time and across projects with different operating risk.

Exhibit 19

Project IRR Assumptions

Project	IRR	Date
Hills Motorway (M2)	17.5%	1994
CityLink	19.0%	1996
Eastern Distributor	16.0%	1996
M5 Motorway	14.0%	1996
M6 Toll	12.2%	1999
M4 Motorway	11.5%	2000
407-ETR	12.1%	Mar-02
WestLink M7	13.5%	Feb-03
Chicago Skyway	10.5%	Oct-04
Connect East	12.1%	Nov-04
Dulles Greenway	12.6%	Sep-05
Indiana Toll Road	12.5-13.5%	Jun-06
Pocahontas Parkway	12.6%	Jun-06

Source: Company Reports.

Application of Debt Capacity Analysis to Measuring Investment Value

To demonstrate how the maximization of debt subject to the constraint that debt is near investment grade can be used in the valuation process, a case study of three power plants is used in same way that three plants were used to evaluate the CAPM. Using the debt capacity framework which involves information on credit rating criteria and assumptions with respect to equity return requirements, the overall cost of capital can be measured and the risks of different investments can be assessed. The case study is used In working through the case, data on the relative debt leverage of projects can be used to measure of value of alternative investments and the resulting project IRR can be used to derive the required project IRR which is analogous to the weighted cost of capital.

The five step approach below allows one to compare evaluate the relative costs and benefits of the natural gas, coal and hydro generating plant investments and determine the implied overall return which is measured as the project IRR. It also allows one to determine relative risks and returns from investing in a plant with a fixed price contract versus a plant that is exposed to market risk. While the approach of using debt capacity shown below is not a simple as plugging three parameters in the CAPM, the debt capacity approach can give much more precise answers with respect to the appropriate WACC and the relative cost of the technologies. The steps to make the risk assessment described below include:

Step 1: Acquire data on DSCR's for plants with similar characteristics as the plant being assessed.
Step 2: Create a project finance model to determine contract rates and debt capacity.
Step 3: Compute the overall return from project IRR implied by contract rates.
Step 4: Evaluate the required merchant prices relative to the contract prices.

Acquiring data on DSCR's for similar projects has already been discussed in the context of contract and merchant power plants above. In actual situations, one may interview bankers to determine key structuring items including the term of the debt, the type of repayment and the minimum DSCR – these are the most important questions to ask your banker as they are selling some exotic financial product. This information is needed in order to determine the amount of debt a project can support. The required equity IRR is assumed to range between 10% and 13%. For purposes of the case exercise, assumptions are shown in the table below.

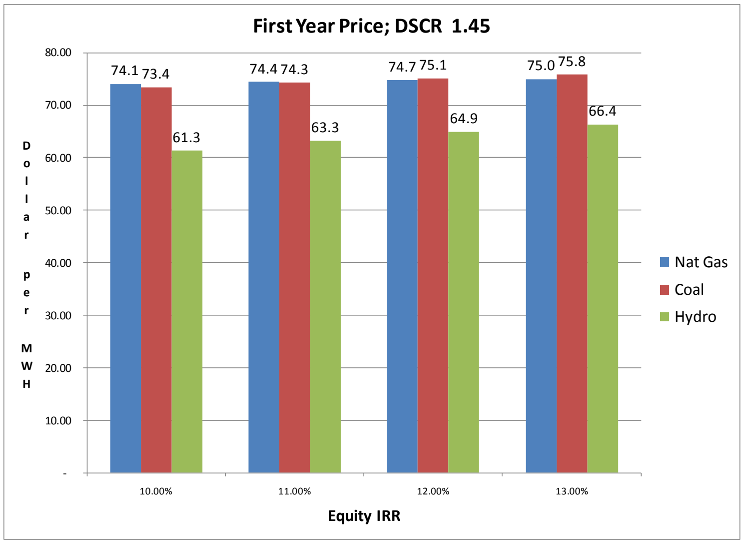
The second step of the process involves incorporating the debt structuring assumptions together with expected operating cash flows to establish a model for the alternative projects. Using the operating cash flow together with debt structuring assumptions and a project finance model, the debt capacity can

be established for an assumed equity IRR and an assumed contract price. This can be established with an optimization algorithm – SOLVER in excel -- that sets both the contract price and the debt leverage subject to the DSCR equaling the target DSCR and also the equity IRR in the model equaling the target equity IRR. The following equations can be put into the solver:

Objective Function:	None
Changing:	Debt Leverage, Contract Prices, Debt Repayments
Constraints:	Target Equity IRR = Actual IRR;
	Target DSCR = Actual DSCR;
	Final year Debt = 0

Alternatively, the NPV of equity cash flow can be computed and set to zero and the repayments can be computed using the formula discussed in chapter 2 with respect to debt sizing. It is a good idea to make equations that go into the solver function as simple as possible. To run multiple scenarios with the solver a data table function does not work and you must create a macro that calls the solver multiple times. Recall that this involves modifying using the tools and references menu in VBA and adding the checking the SOLVER box as well as adding the code USERFINISH = TRUE to the last line of the solver.

Using the 10% to 13% range in equity IRR's, the contract prices for the three plants are shown in the graph below. With a DSCR of 1.45 and a long debt term, the capital intensive hydro plant can result in the lowest price and still generate the required equity IRR. Further, the result is the same for a the range of equity IRR between 10% and 13%. Therefore, unlike the first exercise which used ranges in cost of capital derived from the CAPM and was not able to come up a definitive answer as to which plant is most economic, the range in contract costs using the debt capacity approach are relatively small.



Once the contract prices are computed and the debt capacity is established, free cash flow in the model can be used to measure the project IRR. Since the project IRR corresponds to the assumed equity IRR criteria, it can be used to establish equivalent of the WACC for evaluating the different projects. For example, the range in IRR for the coal plant using a 1.3 DSCR is between 7.46% and 7.84%. The project IRR is similar for the three projects and it ranges by a relatively small amount with the different assumptions for required equity IRR and for the debt term which is assumed to be 25 years for the natural gas plant, 30 years for the coal plant and 35 years for the hydro plant. The range in overall cost of capital is much smaller than the almost 3% range in the CAPM which was between 6.9% and 9.8%.

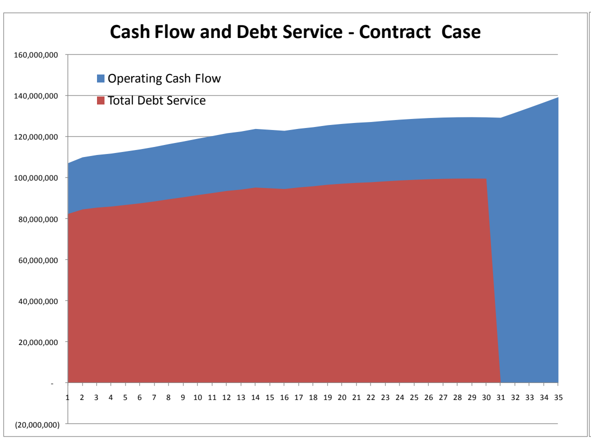
Project IRR with DSCR 1.45			
Equity IRR	Nat Gas	Coal	Hydro
10.0%	8.63%	7.82%	7.72%
11.0%	8.81%	8.01%	7.91%
12.0%	8.98%	8.18%	8.06%
13.0%	9.13%	8.33%	8.20%

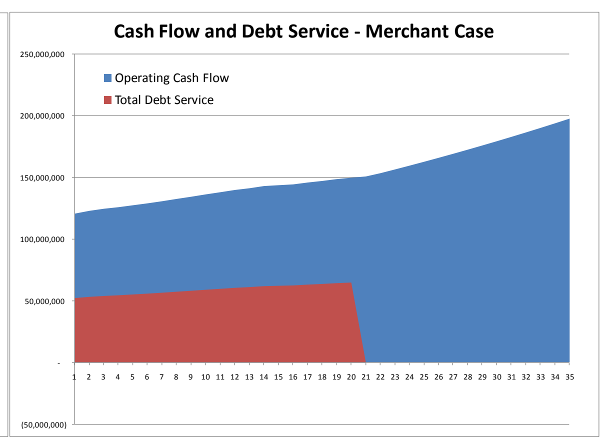
Project IRR with DSCR 1.30			
Equity IRR	Nat Gas	Coal	Hydro
10.0%	8.26%	7.46%	7.39%
11.0%	8.39%	7.61%	7.53%
12.0%	8.50%	7.73%	7.65%
13.0%	8.61%	7.84%	7.74%

In can be argued that the above analysis distorts risk because it assumes a fixed price contract is signed with an off taker. Through signing a contract, the most important risks of demand changes, market capacity and other factors that drive market prices are transferred from the project investors to the off-taker. In discussing the CAPM, recall that the various companies used as a basis for the comparison such as AES and Mirant had plants which own both merchant and contract plants, meaning it is not clear whether the CAPM is primarily measuring risk associated with movements in electricity price. To compute the overall cost of capital associated with merchant plants, the data on DSCR's for plants with merchant exposure can be used to determine the cost of capital for merchant plants. For the merchant case, a DSCR of 2.3 is assumed along with a shorter debt tenor of 15-20 years. The cost of capital for the three plants can be evaluated by using the same operating drivers of value – the capital cost of the plant, heat rate, capacity factor, fuel price, fixed and variable O&M, plant life, and the construction profile – as they were in the contract case. The required merchant price can be computed in the same manner as the contract price and then using the required price, the cost of capital for merchant plants can be established from the project IRR. Ranges in cost of capital are somewhat higher than the contract case as the higher DSCR implies more equity capital and is therefore more sensitive to the equity IRR range. More importantly, the debt capacity method has allowed us to measure the increased risk from merchant price risk relative to the contract prices. For example, the cost of capital for the coal plant assuming a 12% equity IRR increased from about 8% to almost 11%. This sort of analysis that looks at one of items of risk and evaluates it would not be possible using the CAPM model.

Project IRR for Merchant Cases			
Equity IRR	Nat Gas	Coal	Hydro
10.0%	10.68%	9.82%	9.78%
11.0%	11.21%	10.34%	10.32%
12.0%	11.71%	10.81%	10.81%
13.0%	12.19%	11.24%	11.25%

The manner in which lenders require more of a cash flow buffer for increased risk is illustrated on the chart below for the coal plant. In the contract case there is a smaller difference between the top cash flow line and required debt service. Further, because of certainty associated with the term of the contract, the tenor of the debt is also longer. This assessment of risk by lenders is the key driver of the difference in the cost of capital.





In a deregulated market system with more risk borne by investors, less capital intensive plants have a cost advantage as compared to plants that are more capital intensive. This is illustrated on the table below that The difference in risk incorporated in the contract versus merchant cash flow causes a difference in price of $17/MWH or 38%.

Recently, it has become clear that plants without contracts cannot support 60% debt leverage because of financial difficulties of plants in the UK, Argentina, Columbia, the US and other countries. Some analysts suggest that the leverage for a new merchant plant may be as low as 30-40%. Given the change in debt capacity, a similar exercise can be performed which measures the effects of the reduction in the risk appetite resulting from the financial meltdown of merchant capacity. Using debt an assumed debt capacity of 35%, the overall cost of capital increases from 11.5% to 14.2%. Similar exercises could be developed for differences in debt capacity in various countries and for alternative generating technologies.

Summary of Using Debt Capacity to Measure Value

The use of debt capacity from project finance to derive cost of capital and to estimate risk is not perfect. For example, an assumption must be made with respect to the required equity hurdle rate and specific DSCR data on different debt facilities is not very easy to obtain. For projects that will not be financed by debt and there is no debt information, cannot be applied. Further, depends on the judgment of bankers and credit rating agencies which has, to say the absolute least, been questionable. However despite these problems, the debt capacity analysis has allowed us to measure things we could not really evaluate with the CAPM. We have evaluated cost of capital from financial information on specific projects rather than from stock price data on mega corporations with multiple lines of business. We have also been able to isolate risk measurement of specific elements of an asset – contract versus merchant revenues – using financial data that is not produced by management. The approach cannot be boiled down to a simple formula, but the technique relies of assessment by bankers rather than psychological estimates of the EMRP.

Deriving Value from Applying Time Series Analysis and Monte Carlo Simulation to Debt Capacity

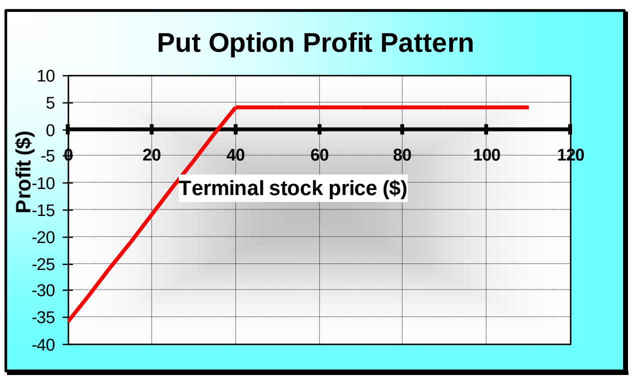
The final part of this chapter extends the debt capacity method through by computing the probability of default, the loss given default and the implied bond rating through developing simulation analysis. This method does not have to rely on arbitrary DSCR criteria and other debt structuring terms to establish the debt capacity which can be non-transparent and subject to errors in judgment. The method works by observing the probability of default that is associated with a low investment grade credit rating and then establishing the debt level that is consistent with that probability of default. One can think of this simulation approach as the same as the last method but using a different basis of information for computing risk and value. Unlike the CAPM and the debt capacity approaches that use stock market and credit analysis information, the time series/debt capacity approach applies statistical parameters that directly measure the risk associated with value drivers such as volatility, mean reversion, price boundaries, price jumps and correlations.

Before discussing any details of the simulation approach one should bear in mind all of the caveats and problems with attempting to apply mathematical analysis to finance. There is no reason to expect distributions of operating cash flow to follow a normal distribution and if a normal distribution is erroneously assumed, the errors in estimating probability of default and debt capacity can be large. Further, random events modeled as jumps can be important in the valuation of debt, but it is impossible to measure the probability and size of extreme events that do not fall into the normal distribution assumptions. Finally, mean reversion and volatility in the long-term equilibrium cash flow can be the

most important factors in a time series equation, but these factors are virtually impossible to measure from historic data, much less forecast. The problems with applying mathematical analysis to credit analysis are highlighted by the Basel II accord that was supposed to reduce the possibility of systematic banking failure through more accurately establishing reserve requirements by more accurately measuring risk. Banks spent billions of dollars coming up with seemingly sophisticated mathematical models to calibrate ratings for specific loans. One does not have to do a fancy study to realize that when massive government funding was needed to save large banks all over the world that Basel II and its associated mathematical models were a complete failure.

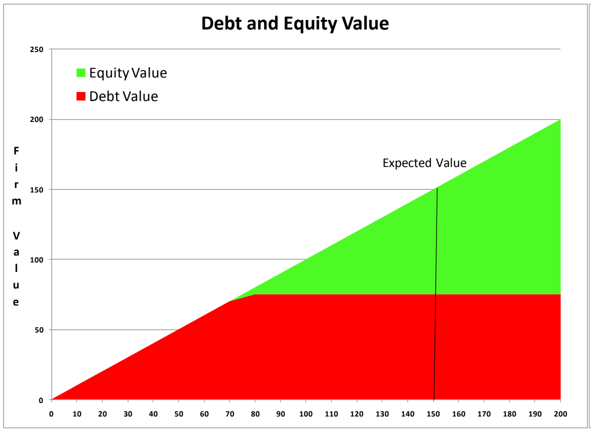
Debt as a Put Option

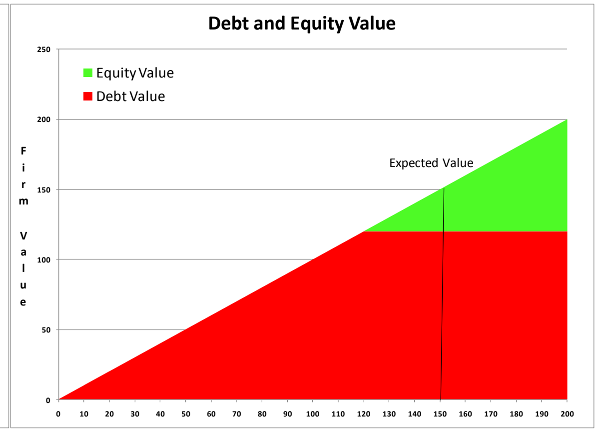
In contrast to traditional credit analysis that focuses on judgment, financial ratios and downside analysis, modern credit analysis is founded on the notion that risky debt has a payoff structure that is similar to the seller of a put option. A put option has no positive value to the buyer if the asset value increases above the strike price, but it allows him to realize a gain when the asset value declines below the strike price. The seller of a put option has an opposite payoff structure as the buyer of a put option. When selling a put option, the seller realizes positive value when the option contract is first sold. The seller does not have to pay anything to the buyer if the asset value remains above the strike price. However if the asset value falls below the strike price, the seller of a put option incurs a loss – he incurs the downside risk of the stock price declining. The accompanying diagram illustrates the payoff structure from selling a put option. The strike price is 40, which means that if the actual value turns out to be below 40, a cash payout is made by the seller of the put to the buyer of the put option. The amount the buyer of the put pays to avoid the downside risk is the premium on the option. This is demonstrated by the straight line part of the graph which is above zero on the graph below.



Issuers of project finance debt are making an investment with a similar cash flow structure as sellers of a put option. If the asset value of the project falls below the value of debt – the strike price – then the put option is in the money. When the put option is in the money, the project has by definition defaulted on the debt contact. The credit spread on debt holders receive when their option is not in the money – when the full debt service is being paid – can be defined as the spread in the interest rate they charge above the risk free rate. This means that if we can use option pricing models to establish the premium that should be paid on a put option, then we can also compute the cost of capital for the project finance debt. The graph below illustrates the payoff patterns to debt and equity as a function of the overall value of the company for two scenarios with different amounts of debt. When the value of the company shown on the x-axis is greater than the value of debt, then the value of the debt is the present value of debt cash flows and the value of value of equity is the remainder of the enterprise value above the value of the debt. The graph on the left shows a case where the value of the debt and the value of equity are about the same if the expected cash flows are realized. If the cash flows are one standard deviation below the expected level, then the value of the enterprise is less than the value of the face value of the debt and lenders experience a loss. The graph on the right shows a case where the value of the equity

is small relative to the value of the debt at the expected level of cash flow. In this case, if one can establish the value of the debt, then most of the enterprise value is also determined.





As a concept, valuation of debt and measurement of credit spreads using option concepts has been around for a long time. For example, Tom Copeland and Vladimir Antikarov note that "Black and Scholes (1973) in their seminal paper recognized that the equity of a leveraged firm is an option on the value of the firm whose exercise price is the face value of the firm's debt and whose maturity is the maturity of the debt."[29] While the notion that risky debt is an option is not new, application of option pricing to measure required yield spreads is controversial in practice. Using option pricing to measure the yield spread on risky debt is difficult to accomplish when a firm has a variety of different projects, many debt facilities and when the corporation has an indefinite life. For a company involved in many lines of business, the volatility parameters are almost impossible to estimate, the maturity structure of many different debt issues makes a strike price difficult to find. Furthermore, future management decisions mean the implied strike price (the value of firm debt) can change dramatically, and mean reversion, price boundaries render the Black-Scholes model and other option valuation approaches useless.

Option price models have been applied to measuring credit spread using a model developed by Bob Merton who modified the Black-Scholes model to compute the probability of default and loss given default from inputs including the current enterprise value of the company, the debt leverage, the volatility in operating cash flow and the risk free rate. The model has been applied by a consulting firm named KMV that is now owned by Moody's. An illustration of the model is shown in the diagram below.

While the model is frequently used by large banks, it is controversial because the credit spread changes when the market value of equity changes and because it depends on many assumptions such as normal distribution of cash flow.of Works as long as very simple time series as discussed in prior chapter. Credit spread that derived in last chapter could be computed with an option model. Problem is when add the important other time series parameters. First work through model with KMV. Also discuss the controversy around Bob Merton.

Step by Step Risk Analysis with the Mathematical Approach

The option pricing concepts can be used to derive the credit spreads for different levels of debt leverage if the level of expected cash flow and the volatility is given. The credit spread that is consistent with a marginal investment grade rating then can be used to define the debt capacity rather than the financial ratios that were used in the last section. Once the debt capacity is calculated, then the process of computing the overall cost of capital is similar to the method above where the project IRR defines the

[29] See Copeland, Thomas and Vladimir, *Real Options: A Practioner's Guide*, Texere, page 163.

overall cost of capital if the expected cash flow is just high enough to provide a minimum rate of return on equity required by sponsors. Calculation of the project IRR in this manner can be computed with different time series parameters such as volatility, mean reversion and jump processes.

The process can be separated into the following steps using a simple random walk process without jumps and without mean reversion:

Step 1: Estimate the expected cash flow and volatility and determine the debt structure
Step 2: Run Monte Carlo Simulations with different levels of debt and compute the credit spread
Step 3: Establish the Debt Capacity from an evaluation of the minimum credit spread
Step 3: Compute the equity IRR and the project IRR using the expected cash flow
Step 4: Adjust the cash flow so as to achieve the target equity IRR
Step 5: Re-compute the credit spread, debt capacity and the project IRR at new level of cash flow

To begin the process, the expected cash flow for the hydro plant in the merchant plant scenario above is used. Debt issued to finance the investment is assumed to have a mortgage structure and a term of 20 years and a credit spread of 1.75% which is used as the assumed credit spread for a marginal investment grade rating. Before running any simulation, a model that has provision for measuring the loss to lenders in downside scenarios must be established. The amount of debt that cannot be paid from free cash flow is the basis for measuring the probability of default and the loss given default. The measurement can be accomplished by creating a project finance model that is structured so as to measure the potential loss on debt after a default has occurred. The loss percent is computed as the present value of the default at the end of the life of the project divided by the aggregate amount of debt issued to finance the project. If the prices are low enough in a year to cause a default on debt service, positive cash flows in future years should be applied to make up the previously accrued debt service on a bond.

The financial model is set up as described in chapter 3 with a model that computes the default on debt and then the loss on debt as a percent of the total debt. The Merton model discussed above cannot be easily used in this analysis because the enterprise value of the project is not known in advance which is in turn due to the fact that the cost of capital has not yet been computed. Computing the expected loss from default on debt using Monte Carlo simulation requires that a time series model be developed and then be incorporated into the financial model. Earlier chapters described the theory and practice of measuring volatility in prices and other time series parameters. In the initial case applies a volatility assumption of 20% along with a to expected electricity prices assumed in the debt capacity case above. This volatility assumption is applied by adding a volatility index that has an initial level of 1.0 and then moving the index forward using the formula:

$$\text{Index}_t = \text{Index}_{t-1} \times (1+\text{NORMSINV(RAND())} \times \text{volatility factor}) + (1-\text{Index}_{t-1}) \times \text{Mean Reversion}$$

Once the model is built with the volatility index applied to the merchant price, a Monte Carlo simulation can be run to tabulate the average loss on debt. The Monte Carlo analysis of time series produces many different price series. Each of these price paths is used in the financial model to test for a default and to measure the loss given default. In some of the price path scenarios, the operating cash flow is insufficient to pay off the debt and a loss occurs. In other scenarios, the cash flow covers all of the debt and allows cash distributions to be made to equity holders. Using Monte Carlo simulation, probability of default is the number of simulations where the loss on debt is greater than zero divided by the total number of simulated price paths.

The mechanics of adding simulation with different levels of debt leverage are relatively simple. After the model is created that computes the percent loss on debt, a macro can be written to tabulate results from many different simulations. Code can be included in the macro so that each simulation is recorded in a different sheet. Finally, results of the simulations maintained in various different sheets can be summarized to evaluate which level of leverage corresponds to the time series parameters. The technique involves the following steps:

Step 1: Write a macro that contains a FOR/NEXT loop over the number of simulations and tabulates the percent loss on debt and other statistics for each scenario. This is accomplished using a FOR NEXT loop together with the CELLS command. Begin with by writing code that loops over the number of scenarios and then with the CELLS function, keep track of the different statistics by using a row number for the variable as illustrated below:

```
FOR ROW = 1 TO NUMBER_SIMULATIONS

        SHEETS(LEVERAGE).CELLS(ROW,1) = RANGE("PERCENT_LOSS")

NEXT ROW
```

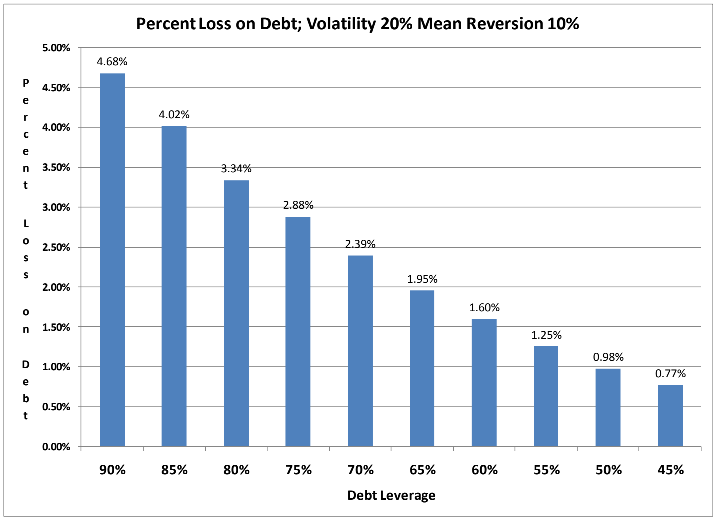
Step 2: Write a second batch macro that adds new sheets for each leverage case and then calls the above each time. This allows the results of each simulation to be maintained in a separate sheet. The two important commands in this macro are the ACTIVESHEET.NAME command and the CALL command as demonstrated below:

```
NUMBER_SIMULATIONS = INPUTBOX("ENTER THE NUMBER OF SIMULATIONS")

LEVERAGE = 95

FOR SCENARIO = 1 TO 10

        LEVERAGE = LEVERAGE – 10
        SHEETS.ADD
        ACTIVESHEET.NAME = LEVERAGE
        RANGE("LEVERAGE") = LEVERAGE/100
        CALL SIMULATE(NUMBER_SIMULATIONS,LEVERAGE)

NEXT SCENARIO
```

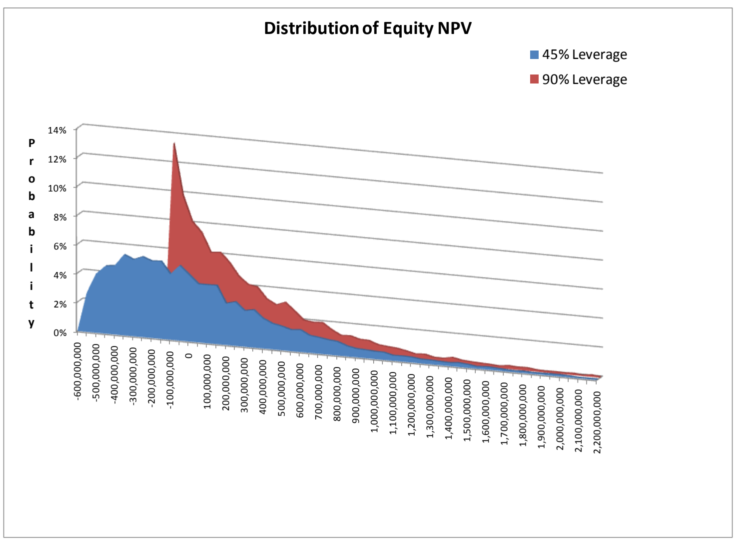
Step 3: Use the INDIRECT command to summarize the results of the different scenarios. The INDIRECT command is useful in summarizing data because you can quickly gather data from other sheets and make calculations with across different numbers of simulations. This works well when you retrieve data from the internet or when you create data in different sheets with a macro. To work with the INDIRECT command, you should begin with the AVERAGE or some other function and then put the INDIRECT command with an adjustable range name as demonstrated in the formula below:

```
AVERAGE(INDIRECT($H377&"!"&I$375))
```

The loan loss as a percent of debt assuming different levels of leverage is shown in the graph below where the x-axis shows different debt to capital percentages and the y-axis lists the credit spread. The debt to capital ratio that results in a 1.75% loss percent assumed to be consistent with an investment grade rating is between 60% and 65%. This relatively high level of debt relative to the overall level of capital invested implies that the debt covers most of the level of investment and the hurdle rate on capital sponsored by equity investors does not have a dramatic effect on the overall cost of capital.



When working through the different debt levels, the value realized by equity sponsors can be retained as well as the loss on debt. The distribution of the NPV to equity holders for the case with the debt leverage of 45% and leverage of 90% is shown on the graph below. The 90% case has a skewed distribution characterized by a put option as expected. More importantly, it is not obvious as to which distribution is more risky simply from observing the different distribution of possible values. The Modigliani and Miller idea that more leverage creates more variability in returns is not clear when there are high levels of debt.



As with the earlier approach where debt capacity derived from the DSCR was used to establish cost of capital, the debt capacity can be used to derive the overall cost of capital through measuring the project IRR. In this exercise, the overall cost of capital with 63% leverage is 9.96% which produces an equity IRR of 11.4%. With different volatility, mean reversion or other time series parameters, the overall cost of capital increases or decreases as the debt capacity changes. For example, when the volatility is 30% instead of 20%, the debt capacity declines to only 30% in order to achieve the approximate 1.75% loss percent discussed above that is roughly consistent with a bond rating that is just investment grade. The relationship between the percent loss and leverage is shown on the table below. In order to achieve the equity IRR of 11.4% with the lower leverage, the price must increase by 11%. With the price increase, the resulting in a project IRR becomes 11.61%. The analysis has demonstrated that with a 10%

increase in volatility, the cost of capital increases by 1.65%, something that could not be evaluated on any kind of reasonable basis using the CAPM.

Leverage and Loss Percent With 30% Volatility and 10% Mean Reversion	
Leverage	Percent Loss
90%	11.21%
85%	10.57%
80%	9.81%
75%	8.50%
70%	7.77%
65%	6.85%
60%	5.73%
55%	5.10%
50%	4.26%
45%	3.85%
40%	3.17%
35%	2.58%
30%	1.88%
25%	1.46%
20%	1.02%
15%	0.63%
10%	0.43%
5%	0.15%

To further extend the example, consider a case where the mean reversion parameter increases from 10% to 20%, which reduces the risk rather than increases risk. With higher mean reversion, the loss rate declines for the different levels of leverage. The higher mean reversion the risk implies that the leverage increases can increase to more than 90%. With the higher leverage, the price can be decreased by about 10% relative to the original case with 10% mean reversion. The reduced price results in an overall project IRR of 8.29%. The three cases with different are summarized compared in the table below.

Cost of Capital with Alternative Volatility and Mean Reversion				
	Volatility	**Mean Reversion**	**Debt Capacity**	**Cost of Capital**
Case 1	20%	10%	63%	9.96%
Case 2	30%	10%	30%	11.61%
Case 3	20%	15%	92%	8.29%

Summary of Time Series Model

The times series approach allows one to objectively relate time series parameters to the cost of capital, something that is not possible with the CAPM or the debt capacity approach. Time series analysis and debt capacity does offer a simple formula to translate risk into value, but it answers questions that were clearly not possible with the CAPM. However, considering where we started—a model that could not differentiate risks for different technologies; a model that did not quantify the impacts of contracts on risk and a model that certainly could not measure the effects of different time series parameters, these approaches are certainly an improvement. The methods directly conflict with the Modigliani and Miller thesis and they do not boil down a simple formula which attempts to measure risk in terms of standard deviation.

As explained in the introduction however, the approach requires estimates of prospective volatility and other time series parameters that can be very unstable over time, impossible to estimate from historic data. In particular, small changes in the mean reversion parameter can have fairly dramatic effects on debt capacity and the overall cost of capital. When the mean reversion parameter increased from 10%

to 15% the debt capacity jumped from 63% to 92%. If the mean reversion is even higher than the debt capacity is more than 100%.

Appendix 1

Appendix 3
Capitalization Rates in Real Estate

The definition of a capitalization rate is net cash flow realized from a property divided by the net rental. For example, if the value of a property is 1,000 and net rentals for an annual period are 100, then the cap rate is 10%. The inverse of the cap rate is much like the P/E or EV/EBITDA multiples in corporate finance. Many of the ideas in developing the capitalization rate are similar to concepts used in evaluating the P/E ratio or the EV/EBITDA ratio.

If rentals would remain constant for an indefinite period with no growth and no taxes, then the cap rate is simply the overall required rate of return on the property as illustrated below:

Value of Property = Annual Net Rental Cash Flow/Required Annual Return on Cash Flows

Or

Required Annual Rate of Return = Annual Rental Cash Flow/Value of Property

The presence of growth rates limited property lives and taxes makes the analysis somewhat more complex, but one can still derive the required return from the cap rate if the growth rates (there could be differences between long-term and short-term growth rates). To illustrate the process by which one can derive the discount rate in the presence of taxes, growth and a limited life, consider the following formula:

Value = PV of After Tax Cash Flow at Required Rate of Return,

And

Cap Rate = Pre-tax Cash Flow/Value

Using these formulations, one can back into the annual rate of return. The only issue that is a little difficult is the tax depreciation that depends on the value of property sold in a transaction. Since the value of transaction depends on the tax deductions, but the tax deductions also depend on the value of the transaction, a circular reference must be resolved. This can be accomplished through entering a fixed value for the tax basis and then copying and pasting the derived value to this number until a consistent value is derived. The table below illustrates how different derived costs of capital result from varying short-term growth rates and changes in the cap rate.

Appendix 4
Simulation Analysis of Beta

In this appendix, statistical issues with measuring beta are considered by analyzing the beta statistic that would come out of the statistical analysis if there was really some true underlying beta. This can be accomplished using Monte Carlo simulation introduced in the last chapter. To investigate the issue, one

can begin with a simple model in which stock prices are driven by two factors – the first is changes in the overall market and the second is random movements that are independent of overall market movements. The daily movements in the market can be simulated through a random walk equation, where the volatility is simulated on a daily basis. In excel, the following equation could be used to compute changes in the overall market:

$$\text{Market Index}_t = (1+ \text{Market Index}_{t-1}) \times \text{NORMSINV(Rand())} \times \text{Volatility}$$

Once the market movement is calculated, the daily rate of return on the market can be computed [LN(Market Index_t/Market Index_{t-1})]. Then, after the market return established, the price of the stock can be computed using a two part formula as follows:

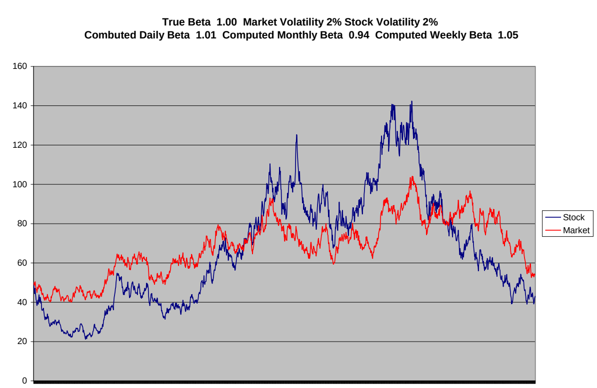
$$\text{Stock Price}_t = \text{Stock Price}_{t-1} \times (1 + \text{Change Due to Market}) \times (1 + \text{Change Due to Random Movements})$$

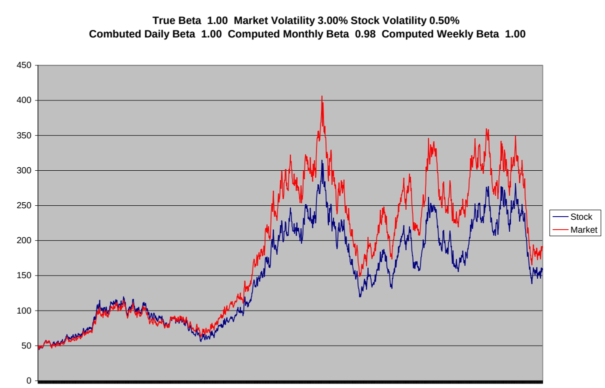
In this formula, stock price movements due to random movements are computed in a similar manner as the market index using the NORMSINV and the RAND functions, while the change related to the market is computed using the equation:

$$\text{Change Due to Market} = \text{Beta} \times \text{Market Index Rate of Return}$$

Finally, after the percent change in stock price is computed on a daily basis, the stock index, the stock price and rates of return are also computed on a weekly basis and a monthly basis. The beta is then computed on the weekly basis and a monthly basis using the SLOPE function in excel.

The accuracy of the estimated beta depends on the size of random movements of the stock price relative to the volatility of the stock index. Examples of simulated series of daily prices for the market index and the stock price with high and low volatility are shown in the two graphs below. The graph on the left shows a scenario where the daily volatility of the stock index is 2% (the annual volatility is 32%) and the daily volatility independent of the market is also 2%. Computed beta for the single series using alternative time frames is shown on the second line of the titles for the graphs. The daily beta is simply computed using the slope function as described above. The weekly and monthly betas are calculated by first establishing a weekly and a monthly series through extracting data at the end of each simulated week or month. Using the weekly or monthly data, the SLOPE function is applied again to compute the alternative betas using the alternative time periods.





Results of simulations for the daily, weekly and monthly betas where the process described above are repeated 5,000 times is shown below. In addition to presenting the average beta across the simulations, the standard deviation is present which demonstrates the variation across the different simulations. To illustrate the variation in betas across the simulations, the graph below the table shows the distribution of betas using different time periods for computing the beta. The simulations are

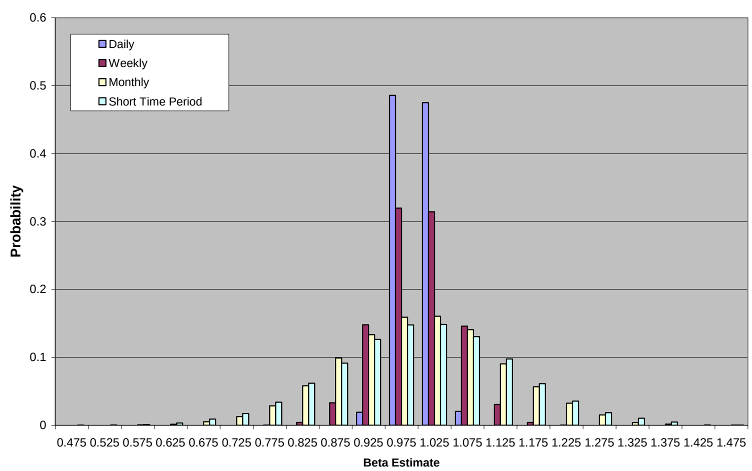
computed using three different levels of beta – 1.0, 0.5 and 2.0 with alternative assumptions with respect to the volatility of the overall market and the volatility of stock independent of the market. Volatility assumptions include three scenarios:

- The assumption that the market has a daily volatility of 2% and the volatility independent of the market is an additional 2%;
- The assumption that the market has a daily volatility of 3% and the volatility independent of the market is an additional 1%; and,
- The assumption that the market has a daily volatility of 1% and the volatility independent of the market is an additional 3%.

Simulation of Betas Using Alternative Time Periods and Alternative Volatility Assumptions

	Daily Regression			Weekly Regression			Monthly Regression			Daily - Short Period		
Daily Volatility of Market	2%	3%	1%	2%	3%	1%	2%	3%	1%	2%	3%	1%
Daily Volatility Independent of Market	2%	1%	3%	2%	1%	3%	2%	1%	3%	2%	1%	3%
True Beta of 1.0												
Average Estimated Beta	1.0000	1.0002	1.0007	0.9992	1.0001	1.0057	1.0010	1.0000	1.0122	1.0020	1.0002	0.9977
Standard Deviation Across Simulations	0.0241	0.0082	0.0738	0.0558	0.0186	0.1636	0.1218	0.0406	0.3656	0.1349	0.0447	0.4048
Average Plus Standard Deviation	**1.02**	**1.01**	**1.07**	**1.05**	**1.02**	**1.17**	**1.12**	**1.04**	**1.38**	**1.14**	**1.04**	**1.40**
Average Less Standard Deviation	**0.98**	**0.99**	**0.93**	**0.94**	**0.98**	**0.84**	**0.88**	**0.96**	**0.65**	**0.87**	**0.96**	**0.59**
True Beta of 0.5												
Average Estimated Beta	0.4997	0.5002	0.5009	0.5003	0.4996	0.5005	0.4972	0.5012	0.5054	0.4975	0.4992	0.5038
Standard Deviation Across Simulations	0.0242	0.0080	0.0728	0.0547	0.0183	0.1655	0.1227	0.0409	0.3711	0.1340	0.0452	0.3997
Average Plus Standard Deviation	**0.52**	**0.51**	**0.57**	**0.55**	**0.52**	**0.67**	**0.62**	**0.54**	**0.88**	**0.63**	**0.54**	**0.90**
Average Less Standard Deviation	**0.48**	**0.49**	**0.43**	**0.45**	**0.48**	**0.34**	**0.37**	**0.46**	**0.13**	**0.36**	**0.45**	**0.10**
True Beta of 2.0												
Average Estimated Beta	2.0003	2.0001	2.0000	2.0016	1.9996	2.0006	1.9997	2.0014	2.0029	1.9987	2.0002	2.0079
Standard Deviation Across Simulations	0.0243	0.0081	0.0742	0.0550	0.0180	0.1620	0.1235	0.0410	0.3649	0.1355	0.0454	0.4044
Average Plus Standard Deviation	**2.02**	**2.01**	**2.07**	**2.06**	**2.02**	**2.16**	**2.12**	**2.04**	**2.37**	**2.13**	**2.05**	**2.41**
Average Less Standard Deviation	**1.98**	**1.99**	**1.93**	**1.95**	**1.98**	**1.84**	**1.88**	**1.96**	**1.64**	**1.86**	**1.95**	**1.60**

Distribution of Betas with Alternative Regressions



Analysis summarized in the table shows that betas computed on a weekly, daily or monthly basis are unbiased estimates of the true daily beta. This can be seen by inspecting the average estimated beta from the various simulations. The result that the statistically measured beta is an unbiased estimator is the same whether alternative true betas are used and with different levels of market and stock volatility. Given the unbiased estimators, the simulation analysis suggests that from a statistical perspective there is no basis to push to beta toward a level of 1.0 when the true beta differs from 1.0 and implies that

some kind of alternative explanation, there is no basis for the mean adjustment made by Value Line and other services. Using the assumption that the true beta exists on the basis of daily market movements, the standard deviation of the beta computed on the basis of daily market movements has a tighter confidence band than betas computed from weekly or monthly data. This means if the stock price volatility is high relative to the overall volatility of the market, then the shorter periods produce a more accurate statistic as measured by the lower standard deviation across scenarios. However, if the volatility of the market is high in relation to the volatility independent of market movements, then the range of beta is similar whether one uses the daily or weekly or monthly data. For example, assuming a true beta of 2.0, the range within one standard deviation assuming 3% market volatility and 1% volatility independent of the market ranges between 2.01 to 1.99 using daily data; between 2.02 to 1.98 using weekly data; and, between 2.04 to 1.96 using monthly data. If one assumed the true beta were driven by weekly or monthly movements rather than daily movements, the standard deviation may be lower for the monthly series.

In real markets, stock returns do not follow a normal distribution; the true beta may not be constant; there may be a one beta when stocks move by a small amount and a second beta when stocks move by a large amount; and, the beta may only apply to certain movements in the market (such as interest rate moves) and not be applicable for other movements (such as oil price changes). In these cases, one may find that the beta estimate is biased for alternative time periods or that some kind of mean reversion adjustment is necessary. For example, say there is a large movement in a stock price due to random movements unrelated to the market and the true beta of a stock is zero. If this large movement (say from a positive earnings announcement) happens to occur when the overall market is increasing, then the measured beta may be high even if the true beta is zero. On the other hand, if the market happens to be decreasing when the sudden upwards stock movement occurs, then the measured beta will be negative. In this example, the positive or negative beta is a statistical anomaly that has nothing to do with the true beta. To correct the statistical problem one could try to make some kind of mean reversion adjustment so that the estimated statistic would be pushed back to its true level of zero. To test this theory and the possibility of other factors distorting the estimated beta, the following additions have been made to the simulation model above:

- Sudden infrequent jumps in prices are included in the simulation, implying that the distribution is not normal. These jumps are made to both the market index and the individual stock by including an input for the frequency of the jump and the magnitude of the jump. The jump is modeled by adding another random variable to the model and then including the downward or upward jump if the random number exceeds the probability. This process is illustrated by the equation below:

  IF(RAND()>Shock Criteria) Include Jump, Ignore Jump)

- The beta is assumed to be subject to random variation rather than constant in the simulation. When including volatility, the beta is assumed to move in a random walk manner. This is modeled in the same manner as the market movement and is modeled using the following formula:

  $Beta_t = (1 + Beta_{t-1}) \times NORMSINV(Rand()) \times Beta\ Volatility$

- The beta is assumed to be different for small market movements than for large market movements. For example, if the market price movement is less than 3% per day, then the beta may be assumed to be 1.2; but if the market movement is more than 3%, then the beta is only .2.

Simulations of these more complex factors is summarized below using different assumptions with respect to the true beta, the frequency and amount of jumps, the volatility in the beta and the difference between the two different betas. The general conclusion of these simulations is that simple beta computed from a regression continues to be unbiased and there is no need for any sort of mean reversion adjustment. Further, computing the beta from a simple least squares regression provides an

unbiased estimate of the true beta no matter what kind of assumption is made. The table shows results of a simulation with respect to large and infrequent jumps which is one of the reasons for applying a mean reversion adjustment. The simulation computes betas with and without a jump process that occurs .1% of the time where the stock moves by 30%. This simulation is made where the true beta is assumed to be zero and where the true beta is 1.0. The table below shows that while the jump causes the confidence in beta to be reduced, there is no bias and no basis for an arbitrary mean reversion adjustment.

Simulation of Betas with and without Random Shocks of 30% with .1% Probability

	Base Case			With Shocks		
	Daily	Weekly	Monthly	Daily	Weekly	Monthly
True Beta of 1.0						
Average Estimated Beta	1.000	0.997	0.995	1.003	1.000	0.995
Standard Deviation Across Simulations	0.027	0.056	0.126	0.042	0.090	0.198
Average Plus Standard Deviation	**0.97**	**0.94**	**0.87**	**0.96**	**0.91**	**0.80**
Average Less Standard Deviation	**1.03**	**1.05**	**1.12**	**1.04**	**1.09**	**1.19**
True Beta of 0.0						
Average Estimated Beta	0.005	0.007	0.003	0.006	0.001	0.001
Standard Deviation Across Simulations	0.073	0.086	0.139	0.081	0.113	0.214
Average Plus Standard Deviation	**(0.07)**	**(0.08)**	**(0.14)**	**(0.07)**	**(0.11)**	**(0.21)**
Average Less Standard Deviation	**0.08**	**0.09**	**0.14**	**0.09**	**0.11**	**0.22**

Appendix 4
Cost of Capital Adjustments for Debt Leverage

Using techniques discussed in Chapter 3, the probability of loss and the credit spread can be computed for different levels of debt to capital through assuming different levels of volatility, mean reversion and other parameters. This technique will be applied below to demonstrate errors associated with assuming that the implied cost of equity keeps increasing as levels of debt in the capital structure increases. Assuming an overall cost of capital of 8% that does not change with the capital structure that was used above along with volatility assumptions, the relationship between leverage and cost of capital that is often presented in finance text changes dramatically. (The presentations in finance texts make the surprising assumption that the cost of debt does not change with changes in the capital structure.) When the cost of debt increases with the higher probability of default caused by the higher leverage, the remaining equity in the capital structure looks more and more like a call option with limited downside risk and a large upside potential. At very high levels of debt, the implied cost of equity eventually can have an even lower cost than debt because of the large upside and limited downside. The analysis below does not come from some sophisticated option pricing analysis, but it the simple mathematical result of computing different credit spreads as a function of different levels of debt. The credit spreads at higher levels of leverage derived from the mathematical analysis below roughly correspond to credit ratings declining from AAA to BB resulting in credit spreads shown in the next section. The specific level of credit spreads depends on the volatility, the life of the project, the mean reversion and the expected return. Use of this type of analysis does not in any way imply that credit spreads and credit analysis should be performed with credit analysis; it is simply used to illustrate the fallacy in assuming that cost of equity increases directly in proportion to the leverage.

The manner in which credit spreads are computed for different levels of debt leverage begins with a simple model where the net present value of cash flows from an investment are compared to the level of debt to determine cash flow consistent with the rate of return, the default and the credit spread. A case without volatility and without default is illustrated below:

Simulation of Credit Spreads			Simple Model			
					Cash Flow	Earned
Assumptions			0	1-Jan-11	(1,000.00)	Spread
Investment	1,000.0		1	1-Jan-11	129.73	-
Cash Flow	129.73		2	1-Jan-12	129.73	-
Volatilty	0%		3	1-Jan-13	129.73	-
Leverage	100%		4	1-Jan-14	129.73	-
Debt	1,000		5	1-Jan-15	129.73	-
Risk Free Rate	5%		6	1-Jan-16	129.73	-
Credit Spread	0%		7	1-Jan-17	129.73	-
Cost of Debt	5%		8	1-Jan-18	129.73	-
Overall Return	8%		9	1-Jan-19	129.73	-
Life	12.00		10	1-Jan-20	129.73	-
			11	1-Jan-21	129.73	-
Results			12	1-Jan-22	129.73	-
IRR Realized	8.00%		13	1-Jan-23	-	-
PV of Positive Cash at Cost (	1,131.31	-	14	1-Jan-24	-	-
Default	-		15	1-Jan-25	-	-
Levelization Factor	8.72		16	1-Jan-26	-	-
Levelized Default	-		17	1-Jan-27	-	-
Added Credit Spread	0.00%		18	1-Jan-28	-	-

In the above table, the cash flow is first derived from finding the cash flow level that is consistent with the target rate of return through use of a goal seek (with a macro as discussed in Chapter 2.) Note how the IRR of 8% realized confirms to the overall return of 8% input in the assumptions. Given the cash flows computed from this process, the present of the cash flows at the risk free rate is compared to the total debt (computed as the leverage multiplied by the investment) to determine the total default. After the default is calculated it is converted to a credit spread by spreading the total default over the number of years of cash flow and then dividing the levelized number by the total debt. The earned credit spread is shown in the right hand column to confirm that its present value sums to the total default.

To demonstrate how the credit spread is computed, the table below assumes that random events cause the return to be below expected, yielding a return of 4%. In this case an added credit spread of .49% would compensate lenders for the default (if it could be realized from other cases where a higher return is earned).

Simulation of Credit Spreads			Simple Model			
					Cash Flow	Earned
Assumptions			0	1-Jan-11	(1,000.00)	Spread
Investment	1,000.0		1	1-Jan-11	109.77	4.90
Cash Flow	109.77		2	1-Jan-12	109.77	4.90
Volatilty	0%		3	1-Jan-13	109.77	4.90
Leverage	100%		4	1-Jan-14	109.77	4.90
Debt	1,000		5	1-Jan-15	109.77	4.90
Risk Free Rate	5%		6	1-Jan-16	109.77	4.90
Credit Spread	0%		7	1-Jan-17	109.77	4.90
Cost of Debt	5%		8	1-Jan-18	109.77	4.90
Overall Return	4%		9	1-Jan-19	109.77	4.90
Life	12.00		10	1-Jan-20	109.77	4.90
			11	1-Jan-21	109.77	4.90
Results			12	1-Jan-22	109.77	4.90
IRR Realized	4.00%		13	1-Jan-23	-	-
PV of Positive Cash at Cost (	957.27	42.73	14	1-Jan-24	-	-
Default	42.73		15	1-Jan-25	-	-
Levelization Factor	8.72		16	1-Jan-26	-	-
Levelized Default	4.90		17	1-Jan-27	-	-
Added Credit Spread	0.49%		18	1-Jan-28	-	-

To compute the required credit spread at different levels of leverage, a simulation can be created with different levels of volatility in two steps. The first step is to add a time series equation to the cash flow as described in Chapter 2, and then compute the average required credit spread for many scenarios from Monte Carlo simulation. Next, the required credit spread from this process is added to the risk free

rate which increases the default and the credit spread. The process of running simulations and computing the credit spread can be accomplished with a simple macro that first performs the simulation at the risk free rate and then inserts the credit spread into the debt cost. The macro can run for different levels of leverage by keeping track of the results of different levels of leverage in different worksheets.

The cost of capital applied to decision making is the discount rate applied to free cash flow. Two remaining issues must be resolved to complete the analysis. First, the Betas must be adjusted to remove the impacts of leverage. Second, an inherent tax shield should be reflected in the overall cost of capital formulation. These adjustments are described in Appendix 2. Once data is collected from the equity markets to establish beta, the cost of capital must be converted to an unlevered or all-equity cost of capital that would reflect the risk of cash flows as if no debt leverage existed. With more debt, the covariation of returns of the security with market returns is greater than it would be without leverage – the beta is increased. To obtain a discount rate useful for application to free cash flow, after data has been collected on equity betas using stock market data, the data must then be adjusted to remove the impacts of debt leverage to derive an all-equity cost of capital.

Computation of All-Equity β

This section describes how to use equity betas computed from market data on a sample of firms to measure cost of capital that can be applied to free cash flow. Because of the statistical derivation of beta, the relation between a beta with debt and without debt can be computed using the following formula:

Unleveraged β	=	Equity β * (Market Value of Equity/Market Value of Capital) + β of Debt x (After Tax Debt/Market Capital)

If the β of debt is zero because the yield on debt does not vary with the overall market, then the following formula can be used to make the adjustment to compute the cost of capital assuming no debt leverage. In this case:

Unleveraged β	=	Measured Equity β from Stock * (Market Equity/Market Capital)

Practical application of the formula for adjusting Beta requires using the market value of equity rather than book value, and debt should measured on an after-tax basis. The reason for why market values is that the equity β is derived from market data. If the market value of equity is substantially above the book value, the measured equity ratio – and beta -- is greater than it would be if book values had been used. For example, if the market value of equity is two times the book value, and the debt to capital ratio is 50% on a book basis, the market value of debt to the market value of capital is 67%. Second, the amount of debt in the formula for B should be measured on an after tax basis since the debt costs are deductible for tax purposes and market values are after-tax. This means that the amount of debt should be adjusted by the formula:

After tax debt	=	Debt Value x Percent of PV represented by interest x (1- tax rate).

If the tax rate is 40%, the market value of equity to the tax-adjusted market value of capital becomes 76%.[30]

If companies own assets which have similar risk characteristics as other companies with publicly traded shares, then, once an adjustment is made to remove the effects of leverage, the all-equity beta should in theory be similar for different companies in the industry. When valuing companies using the CAPM, investment banks often compute betas for a set of comparable companies and then use that average beta in measuring cost of capital. The accompanying table illustrates computation of average betas

[30] The market value of debt is somewhat understated in this approach because interest payments are tax deductible, but the principal payments are not.

after adjustment for leverage using information from previous table. For three of the companies with cash flow risk associated with deregulated energy prices, the average all-equity beta is .94. For three utility companies the average all-equity beta is much lower -- only .42.

Adjustments for Interest Expense Tax Deductibility

To compute the value of an investment using free cash flow, the present value of free cash flow must ultimately be computed from discounting cash flows using the cost of capital estimate. After establishing the all equity beta, the risk free rate and the market premium, we are not quite finished. This formula does not account for the tax benefits associated with tax deductibility of interest. This means the cost of capital to be used in computing present value is not quite the same the all-equity cost of capital that results from adding the risk free rate to the all-equity beta multiplied by the risk premium. Adjusting for tax deductibility of interest expense incorporates the notion that companies can use debt to reduce income taxes even if leverage does not affect the value of investment on a pre-tax basis.

Making an adjustment to the cost of capital to reflect the fact that interest is tax deductible is similar to computing interest expense at an after tax rate in a weighted average cost of capital calculation. To incorporate the effects of debt financing in the cost of capital formula, an interest tax shield can be deducted from the all equity cost of capital as follows:

Interest Shield = (Debt/Capital) x (1-tax rate) x interest rate on debt.

Once the interest tax shield is computed, the cost of capital for use in computing the present value of cash flow is the following formula:

Cost of Capital = R_f + $Beta_{all\text{-}equity}$ x R_m – Interest Shield.

Appendix 5
Incentive to Maximize Debt Capacity

Incentive to Maximize Debt Capacity Because of Income Taxes

The most obvious economic incentive to maximize debt is reduction is because of the tax shield created by interest through retained earnings. This incentive is particularly import in the financing of capital intensive assets such as electricity generating plants and it is important where cash is distributed to equity investors in the form of dividends rather than re-invested in the firm through retained earnings. The analysis below demonstrates that the incentive to reduce income tax is significantly important for electricity generating assets and justifies a lot of legal and banking fees that must be incurred in completing a project finance transaction.

The notion that capital structure is not relevant to valuation of an asset assumes no corporate or personal taxes. However, the level of corporate tax rates, personal income tax rates and capital gains tax rates greatly affect valuation. For most countries in the world that have a system of corporate income tax, interest expense is a deduction from corporate taxable income but there is no deduction on corporate tax returns for distributions (dividends) to equity holders. In terms of personal taxes related to financial securities, interest income on bonds and dividends are both taxable in most countries. Increases in value obtained when cash flow is retained in the company for growth rather than being paid out as dividends are taxed at the capital gains tax rate rather than the personal tax rate. If capital gains tax rates are lower than ordinary income tax rates, the effective personal tax rate on equity income will often be lower than the tax rate on bond interest for companies with low dividend payout.

Merton Miller has developed a formula for quantifying the income tax impacts on value to investors from debt financing can be quantified with the following formula that accounts for both corporate and personal income tax rates:

Tax Gains from Leverage = $(1 - ((1-t_c)(1-t_{ps})/(1-t_{pb}))$ (Interest Deductible Debts)[31]

Tax Rates and Gains from Leverage					
Gains from Leverage = (1 – ((1-tc)(1-tps)/(1-tpb)) (Interest Deductible Debts)					
Leverage Gains	Percent Gains from Leverage	Corporate Tax Rate	Personal Tax Rate in Stock	Personal Tax Rate in Bonds	Interest Deductible Debts
$ 400.00	40%	40%	0%	0%	$ 1,000.00
$ -	0%	40%	0%	40%	$ 1,000.00
$ 400.00	40%	40%	40%	40%	$ 1,000.00
$ 400.00	40%	40%	28%	28%	$ 1,000.00
$ 291.67	29%	40%	15%	28%	$ 1,000.00

In this formula t_c is the corporate tax rate, t_{ps} is the personal tax rate on stocks and the t_{pb} is the tax rate on bonds that interest deductible debts are less than the total per value of debt because repayment of debt principal does not shield tax.. Interest deductible debts represent only the portion of debt value that is made up of interest payments (as opposed to repayment of principal) and therefore is tax deductible. The percent debt value that is deductible depends on the maturity of the debt as well as the interest rate. For example, in the case of a $1,000 per value bond with a bullet maturity in thirty years and an interest rate of 10%, interest represents 91% of the bond value while if the debt has a term of ten years, level payments and an interest rate of 6%, interest deductible payments only represent 26% of the bond value.[32]

After both personal and corporate income taxes are taken into account, debt generally still has a tax advantage relative to equity. The accompanying table illustrates the tax shield value from leverage under various different personal and corporate tax rates given the value of deductible debts. Note that tax savings are large even if there is a significantly lower personal tax rate on equity income (15%) than on bonds (28%). In this case the gains from leverage are still 29%. If you are investing in a hydro plant that costs $1 billion, the use of debt can save hundreds of millions of dollars. It should be noted that Millers equation assumes that leverage does not reduce the marginal tax rate to zero. If the debt leverage becomes so high that taxable income is zero or negative, the corporate tax rate declines to zero and the tax savings disappear.

First Year Price Assuming 15% All Equity IRR with and without Corporate Income Tax				
		With Tax ($/MWH)	Without Tax ($/MWH)	Percent Price Increase From Income Tax
Hydro Plant		75.01	49.15	53%
Coal Plant		51.85	39.03	33%
Gas Plant		44.31	37.90	17%

I suggest that capital intensive electricity assets will often be in the situation where the personal tax rate on equity is similar to the personal tax rate on bonds and the savings from issuing debt have the maximum impact. The personal tax rates are similar for debt and equity income because generating plants cannot generally defer income to equity holders through retaining earnings and re-investing cash flow in the corporation. Unlike a software company or a fast growing pharmaceutical company, the return to investors from a generating plant is realized in the form of dividends rather than capital gains. Since cash flows are not re-invested in the corporation, equity income is taxed at the same personal tax

[31] See Miller, Merton, *Ibid*.
[32] See Section 6 of the workbook for these calculations.

rate as bond income rather than the capital gains rate. The long-life of electricity generating assets also means that bonds can have long maturities suggesting that most of the debt value is in the tax deductible interest portion rather than the non deductible repayment portion. Putting all of this together implies that the tax saving scenario from the above is the maximum rate.[33]

The relative importance of tax deductibility of interest ultimately depends on the capital intensity of the investment. The more of the economic value that is represented by capital, the more important are taxes. To illustrate the importance of income tax in valuation decisions for electricity generation assets, return to the model of the hydro, coal and gas plant presented in the last section. Assume that debt is not used to reduce taxes and assume a required equity return of 15%. We run two tax scenarios for each of the three technologies. The first scenario assumes an income tax rate (state and federal) of 40% while the second scenario assumes no corporate income taxes. The accompanying table demonstrates that the increase in required price to justify the investment due to income tax varies between 17% and 53% depending on the capital intensity of the asset. For the more capital-intensive assets such as coal plants, hydro plants, nuclear plants or renewable energy plants, financial strategy that reduces tax is a crucial issue.[34] (One can argue that the true inherent economic costs must be measured from the scenario without income taxes.)

Incentive to Maximize Debt Capacity and Information Content of Project Finance

A basic objective of any corporation is to maximize value for shareholders through increasing the value of shareholder equity. There is a body of finance theory that suggests financial strategy affects share value through providing information signals to investors. To the extent that project finance provides more positive content to investors than other forms of financing, managers of corporations can maximize share price by using project finance to raise money. As with income tax reduction, information signaling provides a strong incentive to maximize debt capacity.

Large corporations are very complex institutions with multiple lines of businesses, thousands of employees, elaborate revenue and purchase contracts, volumes of assets with different cost structures, and intricate transactions that may not appear on the balance sheet. Because of the complexity of corporations, one of the basic problems for companies that sell securities to the public is conveying information on the true value of the company. As illustrated in the extreme by the Enron bankruptcy, investors often do not really know if earnings imply that future cash flow can be realized and they do not know that earnings figures represent what is currently happening to a company. Given the difficulty in gaining information on the validity of using earnings data in assessing value and given problems with earnings data, investors are naturally keen to acquire information from other sources or signals that suggest the value of a company is increasing or decreasing.

If issuance of project finance debt is a positive signal and if companies desire to use project finance debt because of the positive information content, firms will have another incentive to maximize project finance debt capacity. The remainder of this section therefore addresses the question of whether issuance of project finance debt does provide a significant signal to investors. First, I discuss general theoretical considerations associated with whether debt provides positive informative signals to the market. Second, I consider how project finance affects signals associated with earnings per share.

The method of financing an asset yields considerable information to investors because firms may be reluctant to issue equity because new equity issues provide a negative signal to the market. If a company is selling shares to raise equity rather than buying back its shares, signaling theory implies that management thinks stock prices are higher than they should be, and it expects the price to fall. We sell things when we expect the value to fall – buy low and sell high. The negative market signals associated with new public issues has become part of what has become known as the “pecking order theory.” This theory implies that firms will raise capital in a ranked order so as to limit negative

[33] If municipal debt that is exempt from personal taxes can partially fund an asset, as is the case for pollution control components of electricity generation assets, the tax savings can become more than 50% of the debt value.
[34] The computer files that are used in this analysis are described in section6of the workbook.

information. Internal funds are the first choice of funding for a new investment. If internal funds cannot be used, the next best alternative is bank debt followed in order by public debt, convertible securities and finally, as a last resort, common equity.[35] The incentive to limit negative signals arising from equity issues is described as follows by Clifford Smith:

> Suppose that a potential purchaser of securities has less information about the prospects of the firm than management. Assume further, that management is more likely to issue securities when the market price of the firm's traded securities is higher than management's assessment of their value. In such a case, sophisticated investors will reduce their estimate of the value of the firm if and when management announces a new security issue. Furthermore, the larger the potential disparity in information between insiders and investors, the greater the revision in expectations and the larger the negative price reaction to the announcement of a new issue. A new issue is likely to signal management's belief that the firms' outstanding securities are currently overvalued. Recognizing management's incentives to overvalue securities, the market systematically discounts the value of firms announcing new offerings. And this discount is largest in the cases of common stock and convertible offerings because the value of these securities is most sensitive to changes in the expected profitability of the firm.[36]

Large capital projects such as electric generating plants generally require too much capital to be funded with internal cash flow from operations. Therefore, in the case of financing generating plant investments, incentives related to information content signals involve the type of external financing instrument used – public debt, project finance debt, or common equity. In cases where investments cannot be financed with internal funds, the pecking order theory has been refined to suggest that some forms of debt funding provide more information content to investors than other forms. This refinement of the pecking order theory implies that project finance debt is a positive signal and provides management with a strong incentive to maximize debt capacity.

In an article titled *"Are Bank Loans Different?: Some Evidence from the Stock Market",* Christopher James and Peggy Wier assert that bank debt provides a more positive signal than other forms of debt: "to the extent that banks have better information about and thus greater confidence in a given firm's future than outsiders, they would price their loans to reflect this advantage."[37] The basis of this theory is that bankers assess investments in a more careful manner than the general public and therefore that issuance of bank debt has more positive information content than public debt. Other reasons that bank debt provides positive signals include: (1) it is easier to work with breached covenants in the case of bank debt than public debt; (2) bankers closely monitor the firm after the debt is issued and (3) bank debt has tailored covenants.

Positive investor signals resulting from use of project finance is consistent with the theory that bank debt provides better signals as to the future prospects than public debt. Institutions that provide project finance debt devote considerable resources to evaluating risks including hiring specialized engineers, lawyers and consultants to evaluate the project. Project debt contracts generally include highly specialized financial ratios to monitor credit risks such as the loan life coverage ratio and customized debt service coverage ratios. After the commercial operation date of a project, financial institutions often become highly involved in monitoring the on-going credit of projects – for example operating budgets and capital expenditures must sometimes be approved by bankers. In sum, if use of bank debt provides positive signals to investors because highly qualified outside institutions must sign of on detailed risks, the same is certainly true for project finance debt.

Event Studies

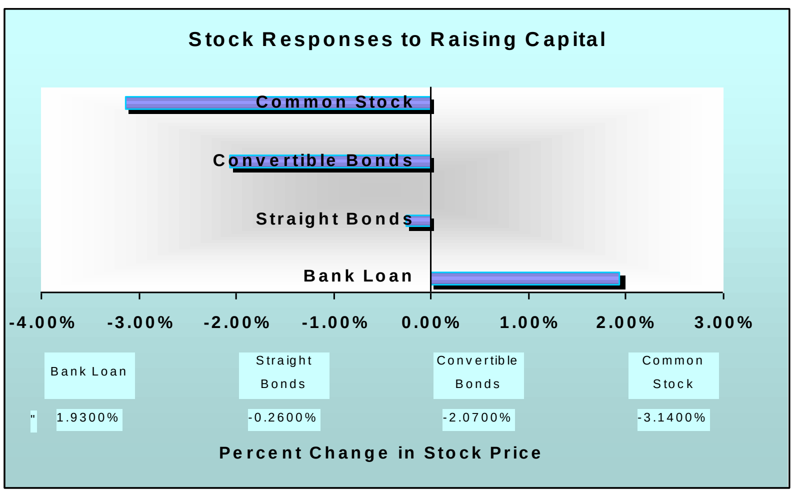
[35] The pecking order theory is analogous to a theory put forth that leverage affects management incentives to create value such as in a leveraged buyout situation or in the case of a firm in financial distress. The idea is that the discipline imposed by bankers causes management to limit unnecessary expenses and improve efficiency.

[36] See Smith, Clifford, "Raising Capital: Theory and Evidence" , Journal of Financial Economics, 1986.

[37] See James, Christopher and Wier, Peggy, The New Corporate Finance, McGraw Hill, page 337.

Without empirical support, a theory is no more than musings in a self-help book or the opinions of a television talk show commentator. Modigilani and Miller used arbitrage theory as well as statistical studies to demonstrate their thesis, the CAPM has been evaluated in numerous complex studies of stock market data and the Black-Scholes option pricing model has been tested by measuring implied volatility suggested by the model as compared to actual volatility. Empirical support of information signaling theories has been investigated using a statistical technique called event studies. This method evaluates how stock prices move relative to other stock prices when a defined event happens. For each event, the stock price moves at the time of the event are accumulated over many different occurrences. A prominent example of the event study technique is analysis of how the value of acquirers and acquires changes upon the announcement of a merger.

Event studies that test information signaling theories tabulate the stock price moves at the time of the financing event are recorded. The accompanying chart summarizes results from various different event studies that measure the stock price impacts of financing.[38] The chart shows that returns have been negative 4% for the issuance of stock and positive 2% for bank debt. While event studies have not been developed specifically to test market reaction to project finance debt, to the extent that project debt is analogous to bank debt, the studies confirm the incentive to maximize debt capacity and minimize the issuance of new equity. I suggest that the event studies summarized in the graph provide empirical evidence positive market signaling for project debt.



While financing can have important effects on stock price, the most significant signal that affects the market value of a company is the level, growth and expectations of earnings per share. When earnings expectations are not met, value generally declines by much more than the impact of the one time earnings reduction. Apparently, the missed earnings estimate is a signal that future prospects are bad. Missing earnings estimates by a few cents per share for one quarter can lead to a reduction in value of ten to twenty-five percent on a $40/share stock. Similar negative signaling occurs when earnings forecasts are reduced.

Even if a large capital project is very profitable over its life, the project can have a negative impact on earnings because of depreciation and deferred tax accounting. This negative earnings “drag” can be mitigated through use of project finance debt. As with the positive signal related to financing, investor signals related to earnings per share provide a strong incentive for managers to maximize the use of debt.

[38] The data has been taken from Clifford Smith and the Christopher James articles referred to above.

Large capital investments negatively affect earnings because of the number of new shares that must be issued in order to raise equity and depreciation charges on the project. Recall the carrying charge discussion from Chapter One. The reason that carrying charges were far higher in the early years of an investment using the "traditional method" is the direct result of this accounting. Through issuing project finance debt instead of equity, the number of new shares is significantly reduced and income taxes are reduced thereby increasing earnings. Limiting the number of new shares issued through investing in large capital projects may not be relevant from a pure theoretical perspective, but it does matter to the stock price. The positive signaling affects and the tax reduction provide very strong incentives for firms to maximize debt capacity.

Appendix 6
Computation of Credit Spreads from Probability of Default and Loss Given Default

PV of Cash Flows(Risk Free Debt) = PV of Cash Flows(Risky Debt)

This implies that the yield on risky debt should be the same as the yield on risk free debt as demonstrated by the formula below where the value at maturity is defined by the yield on bonds in situations where the debt is paid in full. It is useful to begin with a zero coupon bond as follows:

Yield on Risky Debt = Face Value/Current Value of Debt – 1

Credit Spread = Yield on Risky Debt – Risk Free Rate

Face Value/Current Value of Risky Debt = Value at Maturity/Current Value of Risk Free Debt

Once the value

Realized Value = Value of Risky Debt when Debt Does not Default +
Value of Risky Debt when Debt Defaults

Realized Value = (1-Probability of Default) x Face Value +
Probability of Default x Value if Default

Realized Value = (1-Probability of Default) x Face Value +
Probability of Default x Loss Given Default x Face Value

Using abbreviations PD for the probability of default and LGD for the loss given default, the formula can be expressed as:

Realized Value = (1-PD) x Face Value + PD x LGD x Face Value

Realized Value = Face Value x (1-PD + PD x LGD)

Face Value of Risk Free Debt x e^{-rt} = (1-Probability of Default) x Risk Free Debt Value +
Probability of Default x Loss Given Default x Risk Free Value

Assumptions

Risk Free Rate 2	10%
Prob Default 2	2.00%
Loss Given Default 2	100%

Alternative Computations of Credit Spread

Credit Spread 2	2.24%
PD x LGD	2.00%

Proof

	Opening		Prob	Closing	Value
Risk Free	100	→		110.00	110
			Prob	Closing	Value
Risky - No Default	100		0.98	112.24	110.00
Risky - Default	100		0.02	-	-
Total Value					110.00

Credit Spread Formula

Without LGD

(1+rf) = (1-pd)*(1+rf+cs)
(1+rf)/(1-pd) = (1+rf+cs)
(1+rf)/(1-pd)-rf = (1+cs)

cs = (1+rf)/(1-pd)-rf -1

Assumptions

Years	5	BB	5
Risk Free Rate 1	5%		7
Prob Default 1	20.8%	PD	20.80%
Loss Given Default 1	80%		

Alternative Computations of Credit Spread

Credit Spread 1	3.88%
PD x LGD 1	16.64%

Proof

	Opening		Prob	Closing	Value	
Risk Free	100	→		127.63	127.63	
			Prob	Closing	Value	
Risky - No Default	100		0.95	153.01	145.36	
Risky - Default	100		0.05	30.60	1.53	
Total Value					146.89	FALSE

Credit Spread Formula

With LGD

cs = ((1+rf)/((1-pd)+pd*(1-lgd))-rf)^(1/years)-1